Five Basic Elements of Cooperative Learning *

Facilitating effective small group learning means helping group members perceive the importance of working together and interacting in helpful ways. This can be accomplished by incorporating five basic elements into small group experiences. Ultimately, these elements become tools for solving problems associated with groupwork.

Positive Interdependence
When all members of a group feel connected to each other in the accomplishment of a common goal. All individuals must succeed for the group to succeed. (Refer to Chapter 6 for further information.)

Individual Accountability
Holding every member of the group responsible to demonstrate accomplishment of the learning. (Refer to Chapter 7 for further information.)

Face-to-face Interaction
When group members are close in proximity to each other and dialogue with each other in ways that promote continued progress.

Social Skills
Human interaction skills that enable groups to function effectively (e.g., taking turns, encouraging, listening, giving help, clarifying, checking understanding, probing). Such skills enhance communication, trust, leadership, decision-making, and conflict management. (Refer to Chapter 8 for further information.)

Processing
When group members assess their collaborative efforts and target improvements. (Refer to Chapter 9 for further information.)

* See: Johnson, D.W., Johnson, R.T., & Holubec, E.J. (1990). *Cooperation in the Classroom* (rev. ed.). Edina, MN: Interaction Book Company.

Ways to Structure Positive Interdependence *

1. Goal ______ Common purpose is established. One achieves if all achieve.

2. Incentive ______ All teammates receive the same reward if every teammate succeeds.

3. Resource ______ One set of shared materials per group.

4. Role ______ Each member is assigned a complementary and interconnected role.

5. Sequence ______ Overall task is divided into sub-units and usually performed in a set order.

6. Simulation ______ Teammates work through a hypothetical situation to succeed or survive.

7. Outside Force ______ Groups compete against an outside force.

8. Environmental ______ Group members are bound together by the physical environment.

9. Identity ______ Teammates establish a mutual identity through a group name, flag, motto, song, etc.

* See: Johnson, D.W., Johnson, R.T., & Holubec, E.J. (1990). *Cooperation in the Classroom* (rev. ed.). Edina, MN: Interaction Book Company.

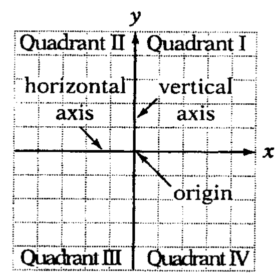
Objective A *To graph points in a rectangular coordinate system............*

POINT OF INTEREST

A rectangular coordinate system is also called a Cartesian coordinate system in honor of Descartes.

Before the 15th century, geometry and algebra were considered separate branches of mathematics. That all changed when René Descartes, a French mathematician who lived from 1596 to 1650, developed **analytic geometry.** In this geometry, a *coordinate system* is used to study relationships between variables.

A **rectangular coordinate system** is formed by two number lines, one horizontal and one vertical, that intersect at the zero point of each line. The point of intersection is called the **origin.** The two lines are called **coordinate axes,** or simply **axes.** Generally, the horizontal axis is labeled the x-axis and the vertical axis is labeled the y-axis.

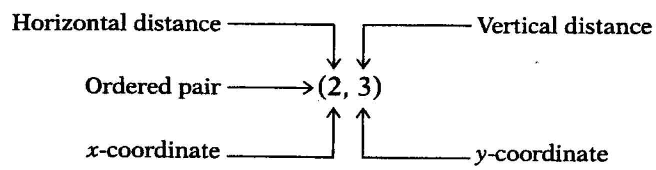


The axes determine a **plane**, which can be thought of as a large, flat sheet of paper. The two axes divide the plane into four regions called **quadrants,** which are numbered counterclockwise from I to IV starting from the upper right.

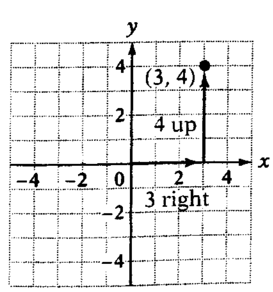
POINT OF INTEREST

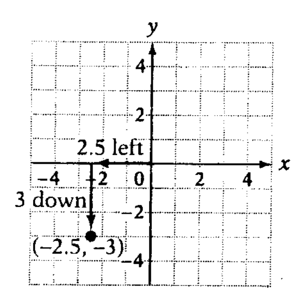
Gottfried Leibnitz introduced the words *abscissa* and *ordinate*. *Abscissa* is from a Latin word meaning "to cut off." Originally, Leibnitz used *abscissa linea*, "cut off a line" (axis). The root of *ordinate* is also a Latin word used to suggest the sense of order.

Each point in the plane can be identified by a pair of numbers called an **ordered pair**. The first number of the ordered pair measures a horizontal distance and is called the **abscissa**, or **x-coordinate**. The second number of the pair measures a vertical distance and is called the **ordinate,** or **y-coordinate.** The ordered pair (x, y) associated with a point is also called the **coordinates** of the point.



To **graph** or **plot** a point in the plane, place a dot at the location given by the ordered pair. The **graph of an ordered pair** is the dot drawn at the coordinates of the point in the plane. The points whose coordinates are (3, 4) and (−2.5, −3) are graphed in the figures below.





Objective B To determine ordered-pair solutions of an equation in two variables

A coordinate system is used to study the relationship between two variables. Frequently this relationship is given by an equation. Examples of equations in two variables include

$$y = 2x - 3 \qquad 3x + 2y = 6 \qquad x^2 - y = 0$$

A **solution of an equation in two variables** is an ordered pair (x, y) whose coordinates make the equation a true statement.

TAKE NOTE

An ordered pair is of the form (x, y). For the ordered pair $(-3, 7)$, -3 is the x value and 7 is the y value. Substitute -3 for x and 7 for y.

➡ Is the ordered pair $(-3, 7)$ a solution of the equation $y = -2x + 1$?

y	$= -2x + 1$
7	$-2(-3) + 1$
7	$6 + 1$
$7 = 7$	

- **Replace x by -3 and y by 7.**
- **Simplify.**
- **Compare the results. If the resulting equation is a true statement, the ordered pair is a solution of the equation. If it is not a true statement, the ordered pair is not a solution of the equation.**

Yes, the ordered pair $(-3, 7)$ is a solution of the equation.

Besides $(-3, 7)$, there are many other ordered-pair solutions of $y = -2x + 1$. For example, $(0, 1)$, $\left(-\frac{3}{2}, 4\right)$, and $(4, -7)$ are also solutions. In general, an equation in two variables has an infinite number of solutions. By choosing any value of x and substituting that value into the equation, we can calculate a corresponding value of y.

➡ Find the ordered-pair solution of $y = \frac{2}{3}x - 3$ that corresponds to $x = 6$.

$$y = \frac{2}{3}x - 3$$

$$= \frac{2}{3}(6) - 3 \qquad \text{• Replace } x \text{ by 6.}$$

$$= 4 - 3 \qquad \text{• Solve for } y.$$

$$= 1$$

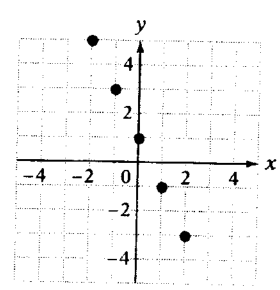
The ordered-pair solution is $(6, 1)$.

The solution of an equation in two variables can be graphed in an xy-coordinate system.

➡ Graph the ordered-pair solutions of $y = -2x + 1$ when $x = -2, -1, 0, 1,$ and 2.

Use the values of x to determine ordered-pair solutions of the equation. It is convenient to record these in a table.

x	$y = -2x + 1$	y	(x, y)
-2	$-2(-2) + 1$	5	$(-2, 5)$
-1	$-2(-1) + 1$	3	$(-1, 3)$
0	$-2(0) + 1$	1	$(0, 1)$
1	$-2(1) + 1$	-1	$(1, -1)$
2	$-2(2) + 1$	-3	$(2, -3)$



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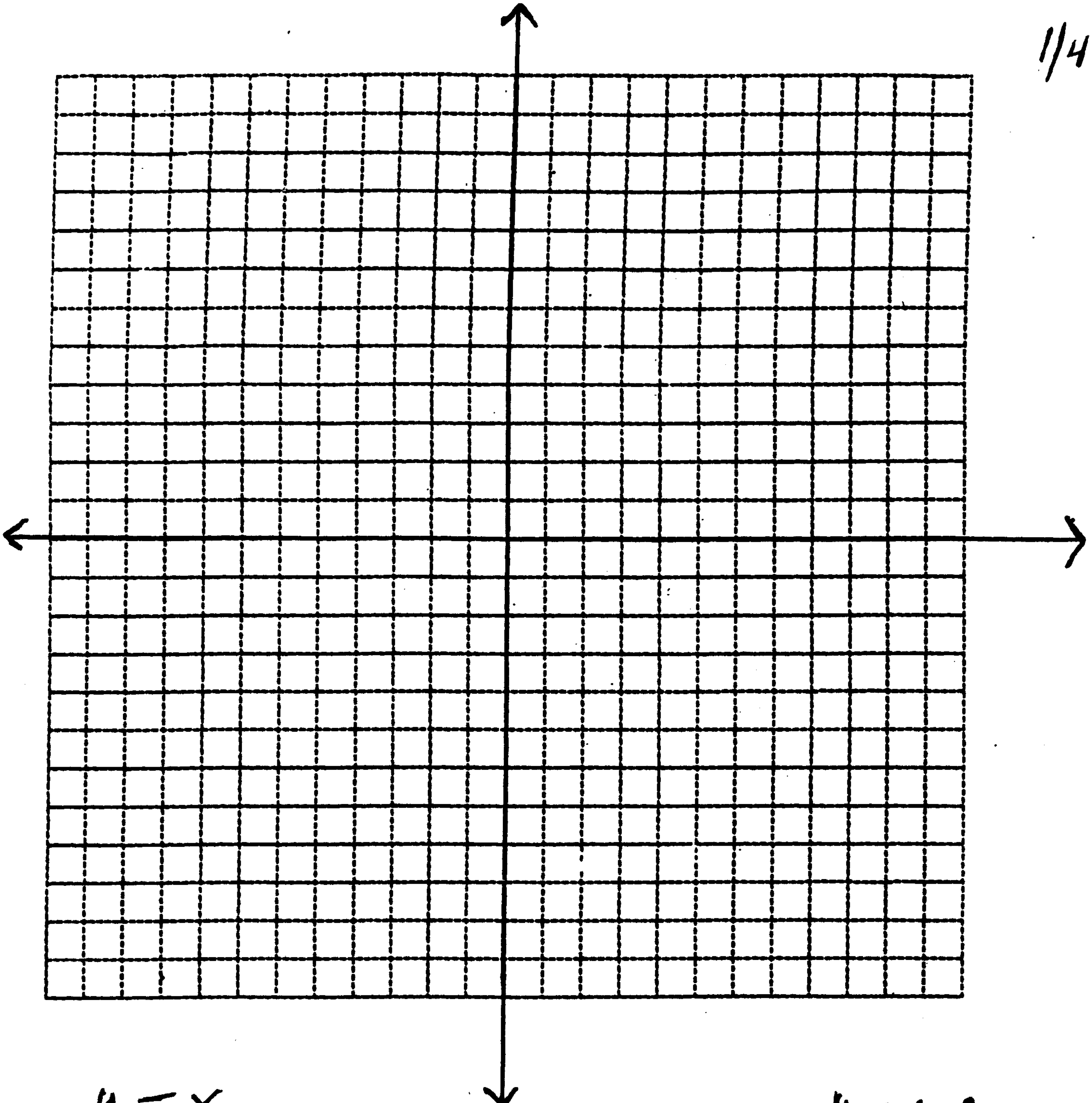


$y = x$

x	y
-1	
-2	
0	
1	
2	

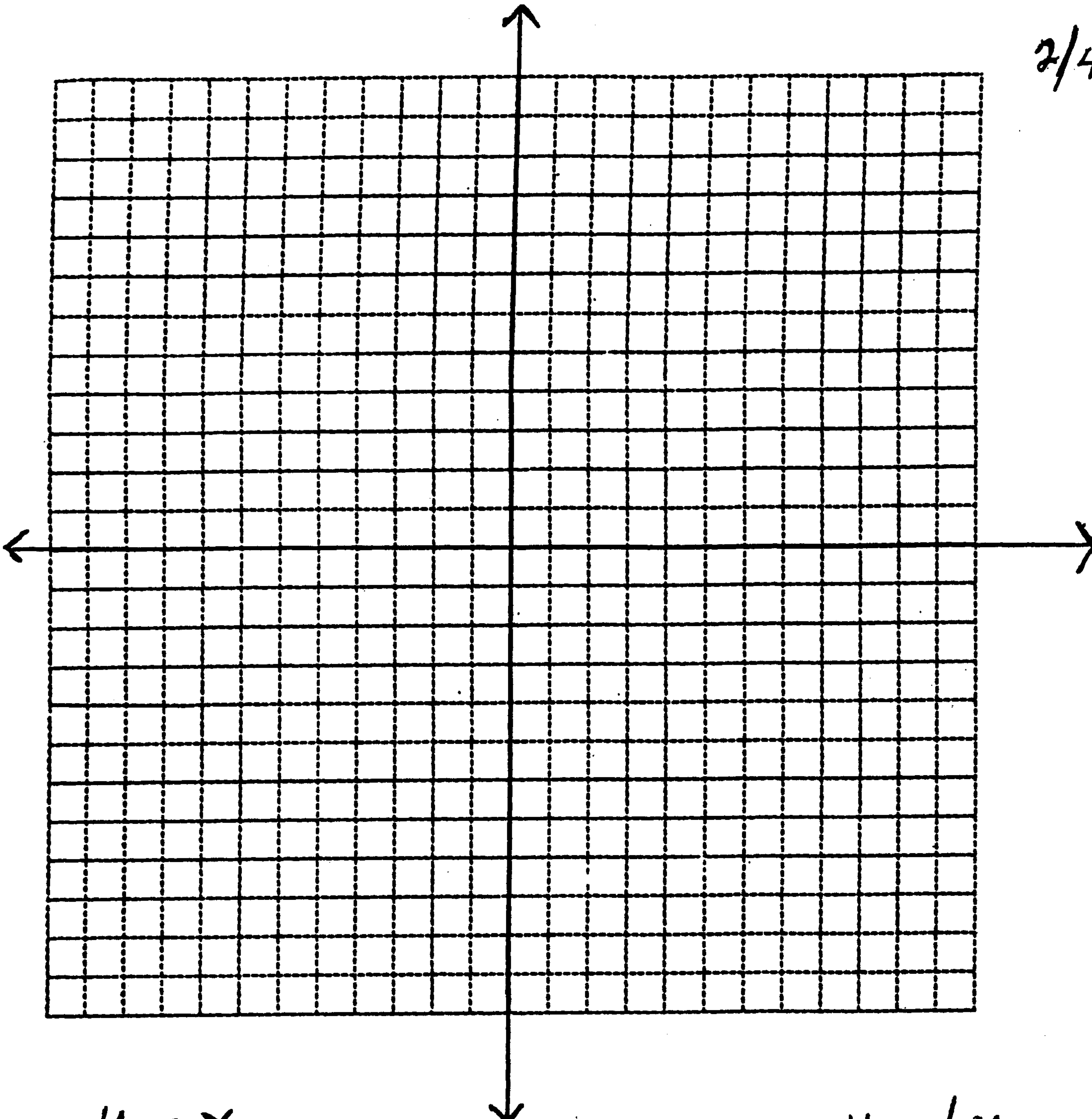
$y = x + 2$

x	y
-1	
-2	
0	
1	
2	

$y = x - 2$

x	y
-1	
-2	
0	
1	
2	

2/4



$y = x$

x	y
0	
1	
2	
-1	
-2	

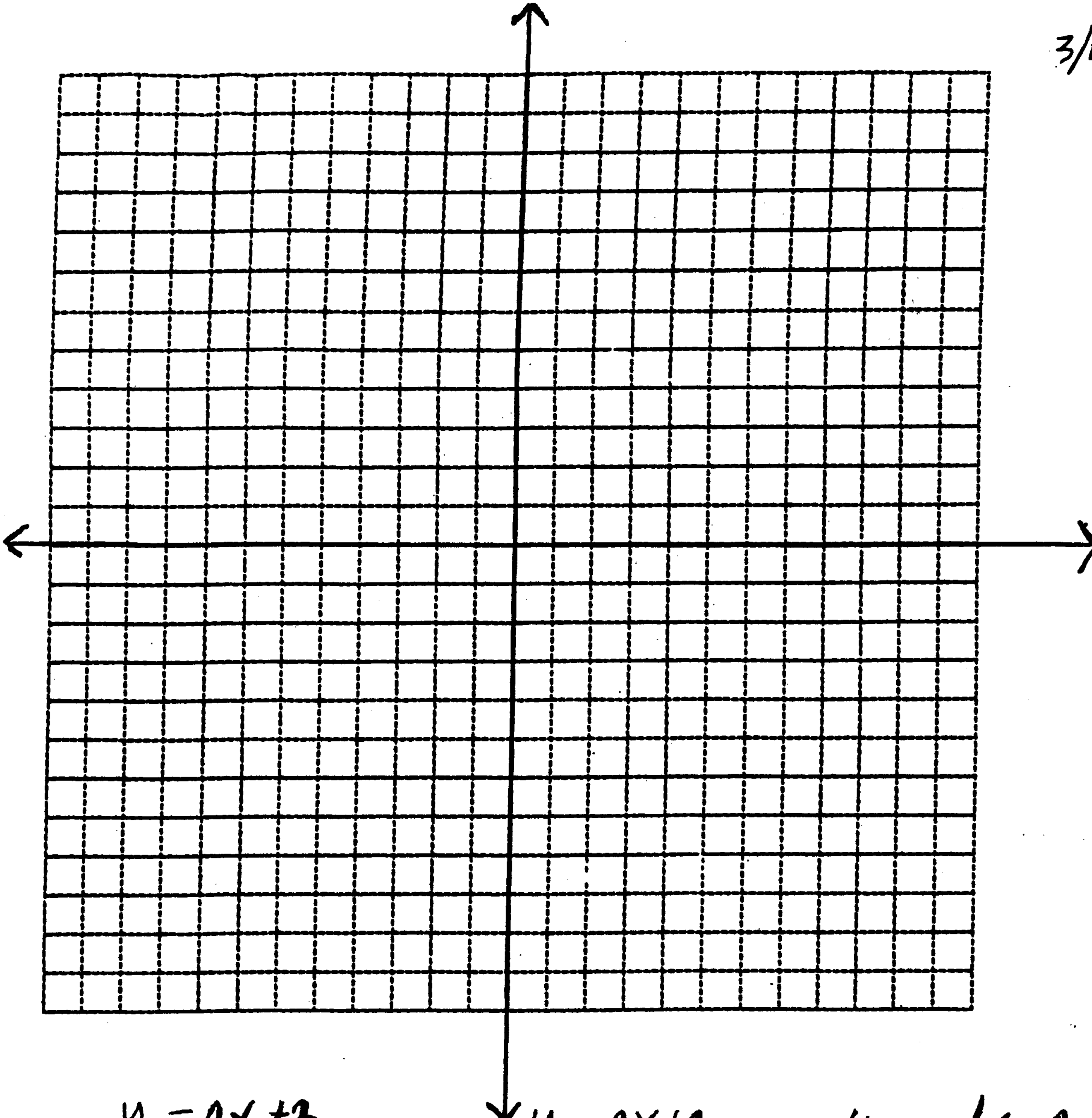
$y = 3x$

x	y
0	
1	
2	
-1	
-2	

$y = \frac{1}{3}x$

x	y
0	
3	
6	
-3	
-6	

3/4



$y = 2x + 2$

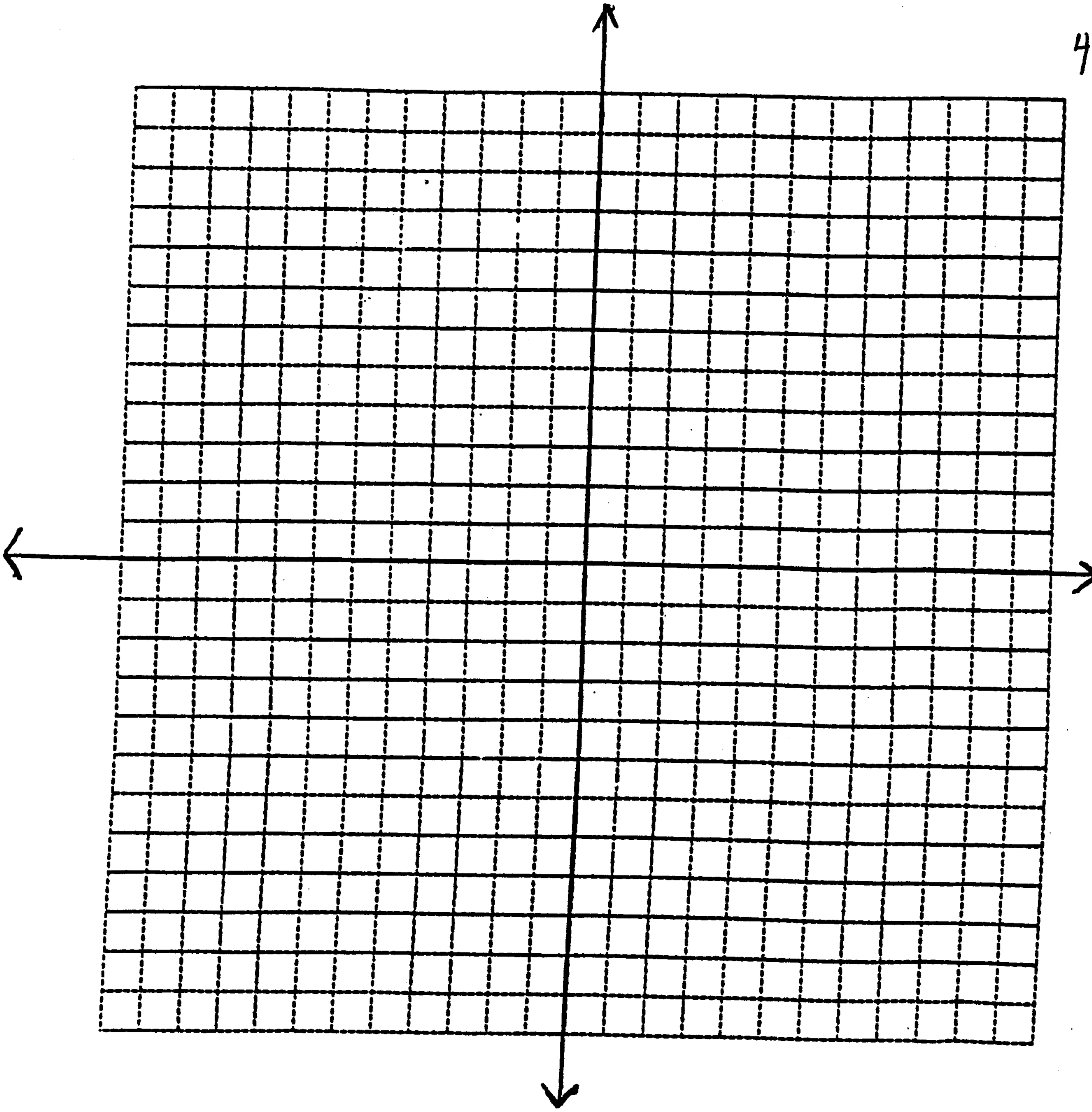
x	y
0	
1	
2	
-1	
-2	

$y = -2x + 2$

x	y
0	
1	
2	
-1	
-2	

$y = -\frac{1}{2}x - 2$

x	y
0	
2	
4	
-2	
-4	



Graph Each equation using the data table provided. You chose the x value + calculate

$y = 2x - 1$

x	y	(x,y)

$y = -3x + 4$

x	y	(x,y)

$y = \frac{3}{2}x - 2$

x	y	(x,y)

LINEAR MODELS

Let us begin our study of models with the discussion of an example that can be represented by a straight line and its corresponding linear equation.

EXAMPLE 2.3

You plan to rent a truck in Canada. The local *Trucks-R-Us* rental company charges $39 (U.S. dollars) per day plus 16¢ per kilometer. Set up a linear model that will describe a one-day rental both algebraically and geometrically. What would be the cost of renting the truck if you expected to drive 100 kilometers? How many kilometers could you drive if you wanted to keep your total costs under $200?

SOLUTION: Since we want to explore a number of different options, we would like our model to express a relationship between kilometers traveled and the total cost of the rental. Using an algebraic approach, let C = total rental cost and K = kilometers traveled. In every case, total cost is computed by adding to the fixed rental fee of $39 an additional 16¢ for each kilometer traveled. This leads to the equation, or model,

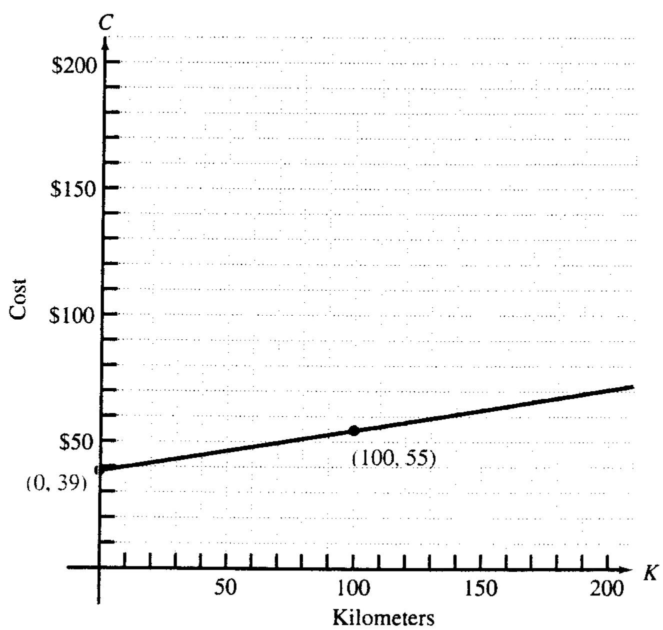
$$C = \$39 + (\$0.16/\text{kilometer}) \cdot (K \text{ kilometers})$$

or, leaving out the units,

$$C = 39 + 0.16K$$

The corresponding graph of this equation is found by plotting points or using the fact that the *C-intercept* is (0, 39) and the slope is 0.16, as shown in Figure 2.10.

Figure 2.10



To find the total cost to drive 100 kilometers, we could *estimate* the value from the graph. The exact solution can be found by substituting 100 kilometers for K (notice from our work with unit analysis that the units simplify correctly):

$$C = \$39 + (\$0.16/\text{kilometer})\,(100 \text{ kilometers}) = \$55$$

To find the number of kilometers you could travel if $200 were available, again we could estimate the value from the graph or substitute $200 for C:

$$\$200 = \$39 + (\$0.16/\text{kilometer})\,(K \text{ kilometers})$$

Solving this equation for K gives (again, observe the units),

$$K = 1{,}006.25 \text{ kilometers}$$

Since we would expect the rental company to charge the full 16¢ for the .25 portion of a kilometer, we should note that $K = 1{,}006$ kilometers would achieve our goal of keeping the total cost for the rental *under* \$200. That is,

$$C = \$39 + (\$0.16/\text{kilometer})\,(1{,}006 \text{ kilometers}) = \$199.96$$

while,

$$C = \$39 + (\$0.16/\text{kilometer})\,(1{,}007 \text{ kilometers}) = \$200.12 \text{ (over the \$200 limit)}$$

■

The solution to the last example brings up a number of points about this particular model. Is it realistic that you would be able to drive over 1,000 kilometers in one day? What if it took you two days to make the trip? Then the model would no longer give an accurate result since this model assumes one day of travel. We will explore these limitations in the exercises.

Example 2.3 developed both an equation and a graph that described the relationship between kilometers traveled and total cost. The steps in determining this mathematical model are summarized in Table 2.1. In essence, the table is a **model of developing a model.**

A Model for Developing a Mathematical Model

1. Choose data that appear to be related.
2. Set up a coordinate system, label axes, and choose appropriate scales.
3. Plot the data points as ordered pairs on the coordinate system.
4. Sketch a **curve** that passes through the points.
5. Determine the equation of the curve.
6. Use the **curve** *and* the equation to predict other outcomes.
7. Consider the reasonableness of your results.
8. Consider any limitations of the model.
9. Consider the appropriateness of the scales.

In practice, the process of developing a model to represent a complex relationship may be very difficult. However, if the data are *linear* the problem is much simpler.

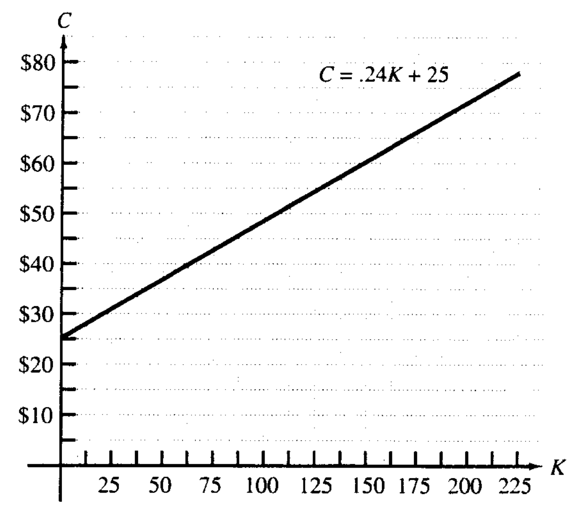
EXAMPLE 2.4 Still planning your move, you call Moving-Mania to see if you can get a better deal. They charge $25 per day plus 24¢ per kilometer. Which company offers a better deal?

SOLUTION: Proceeding in a similar manner to the last example gives the equation

$$C = \$25 + (\$0.24/\text{kilometer})\,(K \text{ kilometers})$$

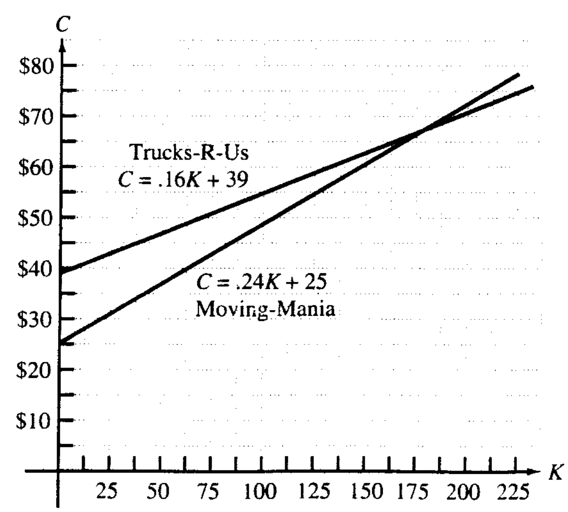
with the corresponding graph shown in Figure 2.11:

Figure 2.11



By drawing the graphs for the two options on the same coordinate system, we see that the lines cross. At the point where the lines cross, the value of C (total cost) and K (mileage) are the same for both models (Figure 2.12).

Figure 2.12



Though it is difficult to determine precisely where the graphs cross, we might *estimate* the coordinate representing miles traveled of that point to be about 170 kilometers, with a corresponding cost of about \$70. To find the precise point at which they cross, we observe that since the costs are equal at the point in question, the two equations for cost must be equal. This time we will write down the equations without the units, though it must be understood that they are still part of the answer. Setting the equations equal to each other, we have

$$39 + .16K = 25 + .24K$$

Grouping together all terms involving K on one side of the equation gives

$$.08K = 14$$

and, upon dividing both sides by .08,

$$K = 175 \text{ kilometers}$$

Using *either* equation we find the cost to be \$67. Notice that both numbers are close to our estimates. Hence, if our trip will cover 175 kilometers, neither company will charge more. This is the **break-even point.** Comparing the graphs, if our mileage is *less* than 175 kilometers, Moving-Mania offers a better deal, since the height of the graph (which represents total cost) is lower than the graph for Trucks-R-Us. If we travel *more* than 175 kilometers, just the reverse is true.

CHAPTER MODULE PREPARATION FORM chapter 1-1

This assignment is due back at the beginning of the next class. I will collect it and check it off during class.

CHAPTER EXPLANATION/SUMMARY

Describe in your own words the objective(s) of chapter 1.1. Explain what the chapter is teaching you.

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____

VOCABULARY BUILDING

List and define in your own words some of the key words or phrases that will help you remember how to solve the chapter exercise problems. For example: "brackets", "symbols", "orders of operations"

Symbols:____

____

____

____

____

____

____

____

____

____

CONCEPTS OR EXAMPLES THAT STILL NEED CLARIFICATION

In this section note any examples or written explanations in the section that you found confusing or that you could not follow the author's explanation. Specify example number and page (i.e. 3a on page 28) or write out the problem in the space below.

Name__________________________________

(over)

WORK OUT THE FOLLOWING PROBLEMS IN ORDER TO PREPARE YOURSELF FOR THE CLASS Show all your steps if it takes several steps to complete the problem. These problems were taken from the chapter exercises. To check your answer find the problem number and check the answer in the back of the book.

Simplify each expression using the correct orders of operations.

1) $13+9\bullet 5$

2) $9\bullet 4-8\bullet 3$

3) $18-2(3+4)$

4) $5\left[3+4\left(2^2\right)\right]$

5) $\dfrac{8+6\left(3^2-1\right)}{3\bullet 2-2}$

6) $\dfrac{4(7+2)+8(8-3)}{6(4-2)-2^2}$

PLEASE WORK ADDITIONAL PROBLEMS PRIOR TO CLASS. The more you complete the better the class will be for you. Use extra paper if you need more room to work out the problems. Hand in this page plus any extra sheets you use to complete the above problems.

ORDER OF OPERATIONS AGREEMENT
STUDY GUIDE

Name ______________________________

In this assignment you will develop a study guide that may be used by your new math buddies and yourself to better understand the idea behind the order of operations agreement.

Write out your own definition <u>in your own words</u> and give several examples that will help someone else understand what is meant by an orders of operation agreement. Use your imagination and have some fun with your definition. Why do we need such an agreement?

Please print of type your guide as we will be sharing our guides during the next class. Use both sides of the paper if needed

Written explanation

EXAMPLES:

(over)

My Very Own Property Definitions

Write out each definition of the specified property in your own words. Try to write each definition in a way that will help you remember the property. Use your imagination! Poetry, hyku, thesaurus wording, stories, anything that will help you remember what is unique about the property and how to apply it. Use the reverse side of the paper if you need more room or use additional paper if you decide to write a novel.

Associative ______________________________

____

____

____

____

Distributive ______________________________

____

____

____

____

Commutative ______________________________

____

____

____

____

Identity property for multiplication ______________________________

____

____

____

____

Identity property for addition ______________________________

____

____

____

____

Inverse property for addition ______________________________

____

____

____

____

Inverse property for multiplication ______________________________

____

____

____

____

Did we leave anything out which needs a definition??

POST TEST REFLECTION NAME______________________________

What mistakes did you make and why?

As a study guide and helpful procedure for preparing for future tests I have prepared a personal review process to help you analyze your mistakes on this test. The intent of this exercise is to help you avoid making similar mistakes later.

In the left hand column write the problem you did incorrectly when you first did the test. In the right hand column write your explanation about why you think you made the original mistake. Then explain how you decided to make your correction(s).

Some reasons for getting a problem incorrect on the first attempt may include:

- **A simple math mistake**
- **Did not remember a formula or procedure**
- **Did not remember a property or rule**
- **Did not use a rule or property correctly**
- **Was not familiar with the problem at all**
- **Transposed numbers during the problem solution or wrote the problem wrong**
- **Any other reason that comes to your mind.**

Question Answered Incorrectly	Reason why you made the mistake

Use additional paper if you need more room.

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***Campus Fax:* (508) 362-8638**
***Administration Fax:* (508) 362-3988**

Dear Elementary Algebra Student,

Welcome to the semester and Elementary Algebra. I can't think of a better way to spend a semester, having fun with algebra. I would like to say hello and offer a few words of advice and encouragement before we meet.. The prerequisite for this course is MAT020- Prealgebra, an identifiable pulse, a temperature of 98.6F and permission from your psychiatrist to subject yourself to this course over the next 15 weeks.

I have a few suggestions that are intended to guarantee your success in this course. If you make a good effort there is no doubt in my mind that you will pass. I am batting 1000 so far. **(What does batting 1000 mean?)**

1. It is vitally, imperatively, critically important that you read the text BEFORE class and TRY TO DO as many exercises as is humanly possible!!

The text is "Introductory/Intermediate Algebra" by Lial, Hornsby and McGinnis 3rd edition
Custom made specifically for elementary algebra at 4C's
The student solutions manual is also required. Please get both books before the first class so you can review the fractions and percents review chapter. This is material covered in MAT020

2. Get extra help immediately if you feel you need it. I am available for extra help 24 hours a day 7 days a week except Sundays from midnight to 6am. I need to sleep sometime. We will work in groups and I will try to arrange study groups outside of class. Tutors will be available also and the math lab is open many hours during the week on a walk in basis.

With all the help available you can't not pass.

3. Back to number 1. The most important thing for you to do is try as many problems before class as possible. That is correct! I am not delirious. But you say "How can I do the problems before they are explained to me??" That is the very essence of this course; to help you gain your math independence. **We will work together in class on the material of the day. By the time the class is over you will know what you are doing.**

You will need plenty of time for homework. If you have a job or family pressures you will need to schedule your hours to allow for blocks of time to study. Experience shows that you need at least 3 hours outside of class for each hour of class. Some people need lots more. I suggest you do about an hour at a time instead of trying to do all the work at once. When you are studying math you can only do so much before you need a break.

I am enclosing a copy of the schedule. Please review the fractions and percent chapter. This is material covered in Basic Math.

I am also enclosing a writing assignment for you to complete before the first class. Bring it with you to the first class. Your math autobiography will help me get to know you better. Please type it.

If you have any questions or concerns about doing algebra in the Fall please feel free to call me at school at 508- 362-2131 x4421. I have an answering machine at school so you can leave a message and I will return your call. You may e-mail me at school at tpanitz@capecod.edu or at home at tpanitz@capecod.net

One last word; if you are concerned about anything I have written here, relax!!!. There are many opportunities for extra help inside and outside of the class. **If you are willing to work hard I can guarantee you will pass this course.**

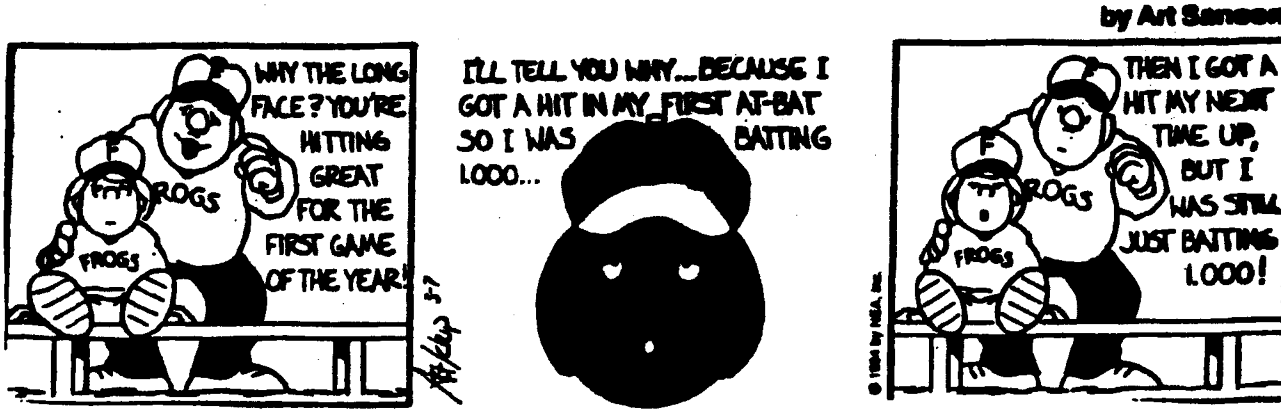
I look forward to meeting you and starting an exciting semester.

Sincerely yours

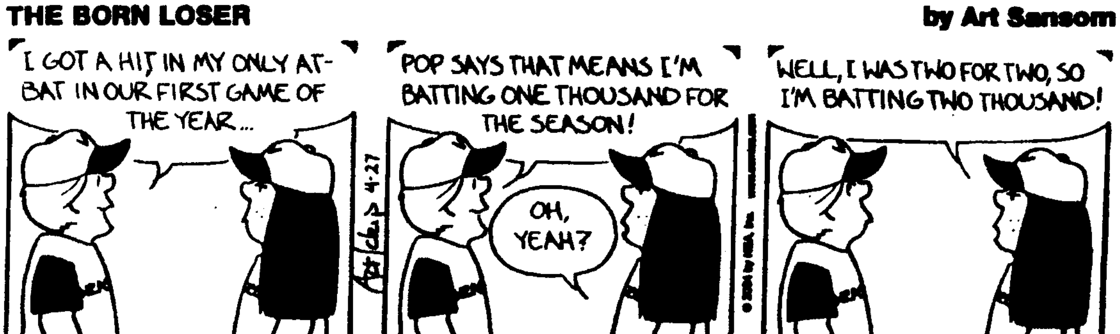
Ted

Ted Panitz

by Art Sansom



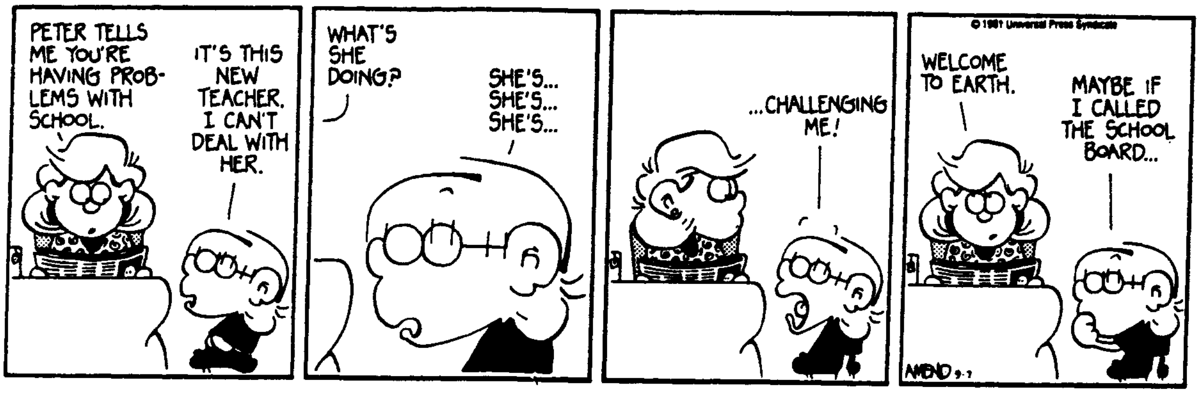
THE BORN LOSER by Art Sansom



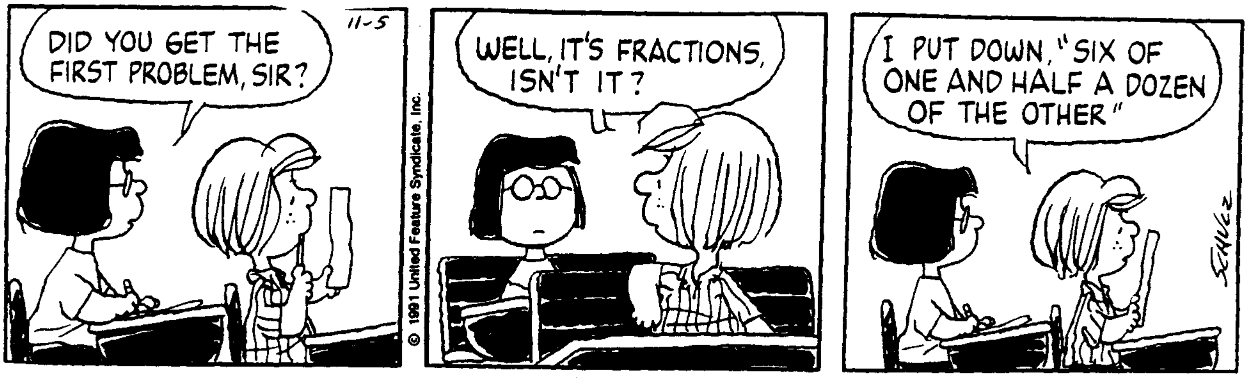
WHAT ARE EFFECTIVE GROUPS LIKE

SOUNDS LIKE	LOOKS LIKE

FOX TROT by Bill Amend



PEANUTS



PEANUTS

PEANUTS