

# Algebra Review

## Absolute value

$$|3| = 3$$

$$|-4| = 4$$

## Signed numbers

To add numbers with the same signs add their absolute values. Give the answer the sign they both have

$$3 + 4 = 7$$

$$(-3) + (-4) = -7$$

To add numbers with unlike signs subtract the smaller absolute value from the larger absolute value. Give the answer the sign of the number having the larger absolute value.

$$7 + (-4) = 3$$

$$-7 + 4 = -3$$

To subtract a signed number, add its opposite

$$\begin{array}{l} 7 - 3 \\ = 7 + (-3) \\ = 4 \end{array}$$

$$\begin{array}{l} 7 - (-3) \\ = 7 + 3 \\ = 10 \end{array}$$

$$\begin{array}{l} -7 - (-3) \\ = -7 + 3 \\ = -4 \end{array}$$

## Multiplying and dividing signed numbers

- if the numbers have the same sign the answer will be positive
- if the numbers have different signs the answer will be negative

$$(-4)(-3) = 12$$

$$\frac{12}{-4} = -3$$

## Order of operations

Do first

Calculations inside Parentheses

Evaluate Exponents

Multiply and Divide working left to right

Add and Subtract working left to right

Do last

$$\begin{array}{l} 9 \div 3 \cdot 5 - 8 \div 2 + 27 \\ = 3 \cdot 5 - 8 \div 2 + 27 \\ = 15 - 8 \div 2 + 27 \\ = 15 - 4 + 27 \\ = 11 + 27 \\ = 38 \end{array}$$

$$\begin{array}{l} \frac{4^2 - 5^2}{(4-5)^2} \\ = \frac{16 - 25}{(-1)^2} = \frac{-9}{1} = -9 \end{array}$$

(1)

## Substitution

Find each value if

$$x=3, y=-4 \text{ and } z=2$$

$$\frac{5x-z}{xy} = \frac{5(3)-(2)}{(3)(-4)}$$

$$= \frac{15-2}{-12}$$

$$= -\frac{13}{12}$$

Place the numbers being substituted in parentheses and then simplify

## Recall The Distributive Property

$$4(x+3) = 4x + 12$$

$$-4(x-2) = -4x + 8$$

$$-(x+3) = -x-3$$

## Recall Like terms

- Like terms have exactly the same variables with the same exponents. They can be simplified using the operations of addition and subtraction

### Like terms

$$3x, 5x$$

$$3x - 5x = -2x$$

$$8x^2, 2x^2$$

$$8x^2 + 2x^2 = 10x^2$$

### Unlike terms

$$x^2, x$$

$x^2 + x$  can not be combined further

## Solving Linear equations in 1 variable

Solve  $5(2x+1)+3=2x+24$

$$10x + 5 + 3 = 2x + 24$$

$$10x + 8 = 2x + 24$$

$$8x + 8 = 24$$

$$\frac{8x}{8} = \frac{16}{8}$$

$$\boxed{x=2}$$

Step 1 Use distributive property to clear parentheses

Step 2 Combine like terms on each side of the equal sign.

Step 3 move variables to the left side of the = sign and numbers to the right

Step 4 Divide both sides by the # in front of the variable (including its sign)

(2)

## Solving Linear equations in 1 variable (continued)

Solve  $6x - 48 = 6$

$$\begin{array}{r} 6x - 48 = 6 \\ + 48 \quad + 48 \end{array}$$

$$6x = 54$$

$$\frac{6x}{6} = \frac{54}{6}$$

$$\boxed{x = 9}$$

### Checking your solution

- if you substitute your solution for the variable in the original equation you should get a true statement

$$6x - 48 = 6$$

$$6(9) - 48 = 6$$

$$54 - 48 = 6$$

$$6 = 6 \quad \checkmark \quad \text{our solution is correct}$$

## Solving Inequalities $<, >, \leq, \geq$

These are solved in a similar fashion to linear equations with one major difference:

If you multiply or divide both sides of an inequality by a negative number the direction of the inequality sign reverses. This usually occurs at the last step if we have a negative number in front of the  $x$ .

Solve.  $2x - 7 \geq 3$

$$+ 7 \quad + 7$$

$$\frac{2x}{2} \geq \frac{10}{2}$$

$$x \geq 5$$

Solve.  $-5(2x + 3) < 2x - 3$

$$-10x - 15 < 2x - 3$$

$$-2x \quad -2x$$

$$-12x - 15 < -3$$

$$+ 15 \quad + 15$$

$$\frac{-12x}{-12} < \frac{12}{-12}$$

$$x > -1$$

Dividing both sides by  $-12$  reverses the direction of the inequality sign.

## Word Problems

Determine what you are trying to find.

Write a Let  $x =$  \_\_\_\_\_ statement

Translate the wording of the problem into an equation.

note  $-3$  times  $x$  would be written  $3x$

- sum means add

- the word "is" means  $=$

- difference means subtract

Solve the equation.

## Simplifying algebraic expressions

Situations involving  
addition and subtraction - combine like terms

$$(3x^2 - 5x - 6) + (5x^2 + 4x + 4)$$

$$3x^2 - 5x - 6$$

$$5x^2 + 4x + 4$$

$$\boxed{8x^2 - x - 2}$$

$$(3x^2 - 5x - 6) - (5x^2 + 4x + 4)$$

$$3x^2 - 5x - 6$$

$$-5x^2 - 4x - 4$$

$$\boxed{-2x^2 - 9x - 10}$$

Multiplication and division use

### The Laws of Exponents

1

$$a^m \cdot a^n = a^{m+n}$$

$$x^3 \cdot x^2 = x^5$$

2

$$(a^m)^n = a^{mn}$$

$$(x^3)^2 = x^6$$

3

$$(ab)^n = a^n b^n$$

$$(3x)^3 = 3^3 x^3 = 27x^3$$

4

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{x^5}{x^2} = x^3$$

5

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$$

6

$$a^{-n} = \frac{1}{a^n}$$

$$x^{-2} = \frac{1}{x^2}$$

7

$$\frac{1}{a^{-n}} = a^n$$

$$\frac{1}{x^{-2}} = \frac{x^2}{1} = x^2$$

8

$$a^0 = 1$$

$$x^0 = 1$$

(4)

Examples - Simplify and write answers with positive exponents

$$(-2a^2b^3)^2 = (-2)^2 a^4 b^6 = 4a^4b^6 \quad \text{by Laws } \boxed{2} \ \boxed{3}$$

$$(3xy^5z^6)(-2xy^3z^{-2}) = -6x^2y^8z^4 \quad \text{by Law } \boxed{1}$$

$$\frac{(2a^{-5}b^4c^3)^{-2}}{(3a^3b^{-7}c^3)^2} = \frac{(2)^{-2} a^{10} b^{-8} c^{-6}}{3^2 a^6 b^{-14} c^6} \quad \text{by Laws } \boxed{2} \ \boxed{3}$$

you must do these first

$$= \frac{(2)^{-2} a^{10-6} b^{-8-(-14)} c^{-6-6}}{3^2} \quad \text{by Law } \boxed{4}$$

$$= \frac{(2)^{-2} a^4 b^6 c^{-12}}{3^2}$$

$$= \frac{a^4 b^6}{3^2 (2)^2 c^{12}} \quad \text{by Law } \boxed{6}$$

$$= \frac{a^4 b^6}{36 c^{12}}$$

Recall "FOILING" First outside inside last

Simplify  $(2x+3)(x-2)$

$$(2x+3)(x-2) = 2x^2 - 4x + 3x - 6 = \boxed{2x^2 - x - 6}$$

Note: the difference between this example and the others above. Here we are multiplying 2 quantities that are composed of a sum or a difference.

In the second example above we were multiplying 2 quantities but each of them consisted of a product.

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# Factoring (rewriting an algebraic expression as a product)

Step 1. Always factor out the greatest common factor  
(if possible) ex.  $3x^2 + 6x = 3x(x+2)$

## Recognize common factoring situations

### Difference of squares

Factor:  $x^2 - 9 = (x+3)(x-3)$

$$4x^2 - 36 = 4(x^2 - 9) = 4(x-3)(x+3)$$

$$64x^4 - 4y^4 = 4(16x^4 - y^4) = 4(4x^2 - y^2)(4x^2 + y^2) \\ = 4(2x-y)(2x+y)(4x^2 + y^2)$$

Note - a difference of squares can be factored but a sum of squares can not be factored

### Trinomials

- factor the first and third term
- figure out the middle term

Factor  $x^2 + 5x - 6 =$   
 $(x+6)(x-1)$

Note:  $x \cdot x = x^2$  the first term  
 $6(-1) = -6$  the last term

$$\begin{array}{c} (x+6)(x-1) \\ \hline 6x \\ -x \end{array}$$

$6x - 1x$  gives the middle term  
 $5x$

Factor  $x^2 - 5x + 6 =$   
 $(x-3)(x-2)$

Note:  $x \cdot x = x^2$  the first term  
 $-3(-2) = 6$  the last term

$$\begin{array}{c} (x-3)(x-2) \\ \hline -3x \\ -2x \end{array}$$

$-3x - 2x$  gives the middle term  
 $-5x$

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## Solve Quadratic Equations

- Step 1. Get zero on the right hand side of the equal sign  
Step 2. Factor \*
- Step 3. Set each factor equal to zero  
Step 4. Solve for the variable

\* If you can not factor the equation and the quadratic is in the form  $ax^2 + bx + c = 0$  then use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve.  $3x^2 - 5x - 2 = 0$

$$3x^2 - 5x - 2 = 0$$

$$(3x + 1)(x - 2) = 0$$

$$3x + 1 = 0$$

$$-1 -1$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$x - 2 = 0$$

$$+2 +2$$

$$x = 2$$

Solve.  $4x^2 - 81 = 0$

$$4x^2 - 81 = 0$$

"difference of squares"

$$(3x - 9)(3x + 9) = 0$$

$$3x - 9 = 0$$

$$3x = 9$$

$$x = 3$$

$$3x + 9 = 0$$

$$3x = -9$$

$$x = -3$$

Solve.  $25x^2 - 6 = 30$

$$25x^2 - 6 = 30$$

$$-30 -30$$

$$25x^2 - 36 = 0$$

$$(5x - 6)(5x + 6) = 0$$

$$5x - 6 = 0$$

$$5x = 6$$

$$x = \frac{6}{5}$$

$$5x + 6 = 0$$

$$5x = -6$$

$$x = -\frac{6}{5}$$

Simplifying Rational Expressions - these are handled the same way we would handle fractions composed of numbers.

Multiplying + Dividing situations

- factor numerators and denominators, cancel if possible before multiplying.
- remember for division - we change to multiplication and flip the second fraction

$$\text{Ex } \frac{6x-18}{3x^2+2x-8} \cdot \frac{12x-16}{4x-12} = \frac{6\cancel{(x-3)}}{\cancel{(3x-4)}(x+2)} \cdot \frac{4\cancel{(3x-4)}}{4\cancel{(x-3)}} = \boxed{\frac{6}{x+2}}$$

Addition + Subtraction situations

- find the LCD (lowest common denominator)
- build the fractions
- add or subtract

$$\begin{aligned} \text{Ex } \frac{x}{x+1} - \frac{2}{x} \\ = \frac{x^2}{x(x+1)} - \frac{2(x+1)}{x(x+1)} \\ = \frac{x^2 - 2(x+1)}{x(x+1)} = \boxed{\frac{x^2 - 2x - 2}{x(x+1)}} \end{aligned}$$

LCD =  $x(x+1)$  Build the Fractions

$$\frac{x}{x+1} \cdot \frac{x}{x} = \frac{x^2}{x(x+1)}$$

$$\frac{2}{x} \cdot \frac{(x+1)}{(x+1)} = \frac{2(x+1)}{x(x+1)}$$

Simplify the expression

$$\begin{aligned} \frac{a}{\frac{a}{x} + \frac{a}{y}} &= \frac{xy \left( \frac{a}{xy} \right)}{xy \left( \frac{a}{x} + \frac{a}{y} \right)} \\ &= \frac{axy}{ay+ax} \\ &= \frac{\cancel{a}xy}{\cancel{a}(y+x)} = \boxed{\frac{xy}{x+y}} \end{aligned}$$

Multiply the numerator and the denominator by the LCD  
LCD =  $xy$



# Lines and Graphs of Lines

an equation with 2 variables demands that each variable behaves in a specific way in relation to the other variable

consider  $y = 2x + 1$  if  $x = 0$  then  $y = 2(0) + 1 = 1$   
 if  $x = 2$  then  $y = 2(2) + 1 = 5$

x	y
0	1 (0,1)
2	5 (2,5)

are points on the line.

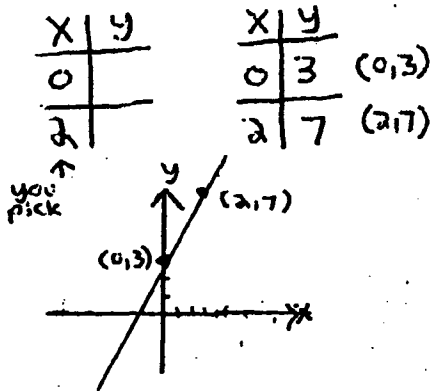
## General Formulas for the equation of a line

$$y = mx + b$$

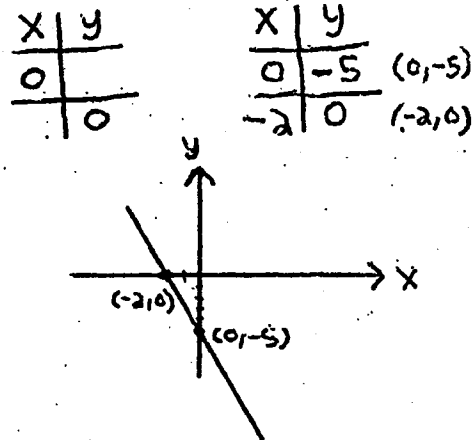
where  $m = \text{slope}$   
 $b = y \text{ intercept}$

$$Ax + By = C$$

Graph  $y = 2x + 3$



Graph  $5x + 2y = -10$

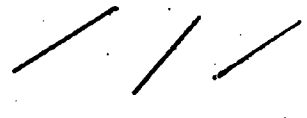


graph by finding x, y intercepts

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

where  $(x_1, y_1), (x_2, y_2)$  are two points we know

Comments about slope - lines with positive slope rise to the right



- lines with negative slope fall to the right

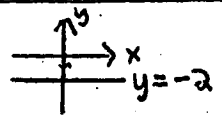


- can be determined from an equation in the form  $y = mx + b$   $m = \text{slope}$

### Special Lines

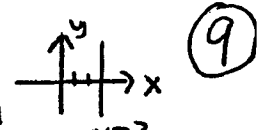
$y = c$   $c = \text{some \#}$

Horizontal line  
 $m = 0$



$x = c$   $c = \text{some \#}$

Vertical line  
 $m$  is undefined



## Solving Systems of Equations

by Elimination step 1 multiply one or both equations so one variable cancels out when they are added together

step 2 Solve the resulting equation

step 3 Find the other variable by substituting the result from step 2 back into either of the original equations.

$$\begin{array}{l} \text{Solve } 3x + 2y = 25 \\ x - 3y = -21 \end{array}$$

$$\begin{array}{l} 3x + 2y = 25 \\ -3(x - 3y = -21) \end{array}$$

$$\begin{array}{l} 3x + 2y = 25 \\ -3x + 9y = 63 \\ \hline 11y = 88 \\ \boxed{y = 8} \end{array}$$

$$\begin{array}{l} 3x + 2y = 25 \\ 3x + 2(8) = 25 \\ 3x + 16 = 25 \\ 3x = 9 \\ \boxed{x = 3} \end{array}$$

Keep in mind that what we are doing here is finding the point where the two lines intersect so the obtained values should satisfy both equations.

- special situations - the two lines are the same line - after you add them together you are left with  $0 = 0$
- the two lines are parallel - after you add them together you are left with a false statement ex:  $0 = 5$  (they never intersect)

## Simplify Radicals

$$\sqrt{81} = 9$$

$$\sqrt{x^2} = x$$

$$\begin{aligned} \sqrt{8} \cdot \sqrt{10} &= \sqrt{80} \quad \left( \begin{array}{l} \text{the largest} \\ \text{perfect square} \\ \text{that divides} \\ \text{evenly into 80 is 16} \end{array} \right) \\ &= \sqrt{16 \cdot 5} \\ &= \boxed{4\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \sqrt[3]{24x^3y^6} &= \sqrt[3]{8 \cdot 3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^3 \cdot y^3} \\ &= 2\sqrt[3]{3} \cdot x \cdot y \cdot y \\ &= \boxed{2xy^2\sqrt[3]{3}} \end{aligned}$$

Perfect squares	Perfect cubes
4	8
9	27
16	64
25	...
36	...
49	...
64	...
81	...
100	...

(the largest perfect cube that divides evenly into 24 is 8)

## Solve an absolute value equality

$$|2x + 2| = 12$$

$$\begin{array}{l} \swarrow \\ 2x + 2 = 12 \\ 2x = 10 \\ \boxed{x = 5} \end{array}$$

$$\begin{array}{l} \searrow \\ -(2x + 2) = 12 \\ -2x - 2 = 12 \\ -2x = 14 \\ \boxed{x = -7} \end{array}$$

There are two cases - the quantity inside the absolute value is positive

- the quantity inside the absolute value is negative

There are two solutions.

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## Simplify - signed numbers

$-4 + (-6)$	$-4 + (-6)$ $= \boxed{-10}$
$7 + (-11)$	$7 + (-11)$ $= \boxed{-4}$
$10 - (-2)$	$10 - (-2)$ $= 10 + 2$ $= \boxed{12}$
$-5 - 3$	$-5 - 3$ $= -5 + (-3)$ $= \boxed{-8}$
$-10 + 4$	$-10 + 4$ $= \boxed{-6}$
$8(-3)$	$8(-3)$ $= \boxed{-24}$
$(-5)(-7)$	$(-5)(-7)$ $= \boxed{35}$
$\frac{-25}{-5}$	$\frac{-25}{-5} = \boxed{5}$
$\frac{-12}{4}$	$\frac{-12}{4} = \boxed{-3}$

# Evaluate - Substitution

$$a + 3b - c$$

for  $a = -3$ ,  $b = 2$  and  
 $c = -1$

$$\begin{aligned} a + 3b - c &= -3 + 3(2) - (-1) \\ &= -3 + 6 + 1 \\ &= \boxed{4} \end{aligned}$$

$$-2x^2 - 6x + 4$$

for  $x = -3$

$$\begin{aligned} -2x^2 - 6x + 4 &= -2(-3)^2 - 6(-3) + 4 \\ &= -2(9) + 18 + 4 \\ &= -18 + 18 + 4 \\ &= \boxed{4} \end{aligned}$$

Find each value if  
 $x = 3$ ,  $y = -4$ , and  $z = 2$

a)  $xyz - 4z$

a) 
$$\begin{aligned} xyz - 4z &= (3)(-4)(2) - 4(2) \\ &= -12(2) - 4(2) \\ &= -24 - 8 \\ &= \boxed{-32} \end{aligned}$$

b)  $2x - y$

b) 
$$\begin{aligned} 2x - y &= 2(3) - (-4) \\ &= 6 + 4 \\ &= \boxed{10} \end{aligned}$$

c)  $x(y - 3z)$

c) 
$$\begin{aligned} x(y - 3z) &= 3(-4 - 3(2)) \\ &= 3(-4 - 6) \\ &= 3(-10) \\ &= \boxed{-30} \end{aligned}$$

d)  $3y^2 - 2x + 4z$

d) 
$$\begin{aligned} 3y^2 - 2x + 4z &= 3(-4)^2 - 2(3) + 4(2) \\ &= 3(16) - 6 + 8 \\ &= 48 - 6 + 8 \\ &= 42 + 8 \\ &= \boxed{50} \end{aligned}$$

# Solving Linear Equations in One Variable

Solve the following for X

$$50 - X - (3X + 2) = 0$$

$$50 - X - (3X + 2) = 0$$

$$50 - X - 3X - 2 = 0$$

$$-4X + 48 = 0$$

$$-48 - 48$$

$$\frac{-4X}{-4} = \frac{-48}{-4}$$

$$X = 12$$

$$8 - 4(x-1) = 2 + 3(4-x)$$

$$8 - 4(x-1) = 2 + 3(4-x)$$

$$8 - 4x + 4 = 2 + 12 - 3x$$

$$-4x + 12 = -3x + 14$$

$$+3x \quad +3x$$

$$-x + 12 = 14$$

$$-12 \quad -12$$

$$-x = 2$$

$$\frac{-x}{-1} = \frac{2}{-1}$$

$$x = -2$$

$$\frac{2}{3}x - 5 = x - 3$$

$$\frac{2}{3}x - 5 = x - 3$$

$$3\left(\frac{2}{3}x - 5\right) = 3(x - 3)$$

$$2x - 15 = 3x - 9$$

$$-3x \quad -3x$$

$$-x - 15 = -9$$

$$+15 \quad +15$$

$$\frac{-x}{-1} = \frac{6}{-1}$$

$$x = -6$$

multiply both sides  
of the equation by 3  
to clear the fractions

- use the distributive  
property

$$\frac{3x}{2} - 5x = 6$$

$$\frac{3x}{2} - 5x = 6$$

$$2\left(\frac{3x}{2} - 5x\right) = 2 \cdot 6$$

$$3x - 10x = 12$$

$$-7x = 12$$

$$\frac{-7x}{-7} = \frac{12}{-7}$$

$$x = \left(-\frac{12}{7}\right)$$

multiply both sides  
of the equation  
by 2 to clear  
the fraction

- use the  
distributive  
property

③

# Solve

$$4y - (2y + 6) = 10$$

$$4y - (2y + 6) = 10$$

$$4y - 2y - 6 = 10$$

$$2y - 6 = 10$$

$$+6 \quad +6$$

$$\frac{2y}{2} = \frac{16}{2}$$

$$y = 8$$

$$\frac{x}{4} - 3 = \frac{x}{2} + 1$$

$$\frac{x}{4} - 3 = \frac{x}{2} + 1$$

$$4\left(\frac{x}{4} - 3\right) = 4\left(\frac{x}{2} + 1\right)$$

$$x - 12 = 2x + 4$$

$$-2x \quad -2x$$

$$-x - 12 = 4$$

$$+12 \quad +12$$

$$\frac{-x}{-1} = \frac{16}{-1}$$

$$x = -16$$

multiply both sides of the equation by 4 to clear the fractions

- We use 4 because it is the lowest common denominator
- the smallest # that both 2 and 4 divide into evenly

$$\frac{7}{x+1} = \frac{3}{x-3}$$

$$\frac{7}{x+1} = \frac{3}{x-3}$$

$$7(x-3) = 3(x+1)$$

$$7x - 21 = 3x + 3$$

$$-3x \quad -3x$$

$$4x - 21 = 3$$

$$+21 \quad +21$$

$$\frac{4x}{4} = \frac{24}{4}$$

$$x = 6$$

when you have an equation with only a single fraction on each side of the equal sign

you can solve by cross multiplying

# Solve Inequalities

$$3x + 4 < 12 - x$$

$$3x + 4 < 12 - x$$

$$+x \qquad +x$$

$$4x + 4 < 12$$

$$-4 \quad -4$$

$$\frac{4x}{4} < \frac{8}{4}$$

$$x < 2$$

$$6 - 2(t + 1) \geq 8$$

$$6 - 2(t + 1) \geq 8$$

$$6 - 2t - 2 \geq 8$$

$$-2t + 4 \geq 8$$

$$-4 \quad -4$$

$$\frac{-2t}{-2} \geq \frac{4}{-2}$$

$$t \leq -2$$

Dividing both sides  
by  $-2$   
reverses the direction  
of the inequality sign

$$3(x - 4) - (x + 1) \leq -12$$

$$3(x - 4) - (x + 1) \leq -12$$

$$3x - 12 - x - 1 \leq -12$$

$$2x - 13 \leq -12$$

$$+13 \quad +13$$

$$2x \leq 1$$

$$\frac{2x}{2} \leq \frac{1}{2}$$

$$x \leq \frac{1}{2}$$

# Word Problems

Four times a number increased by 10 is equal to 34. What is the number?

Let  $x =$  the number

$$4x + 10 = 34$$

$$\quad -10 \quad -10$$

$$\frac{4x}{4} = \frac{24}{4}$$

$$x = 6$$

Samantha is three times as old as Susan. The sum of their ages is 48. How old is each?

Let  $x =$  Susan's age

$3x =$  Samantha's age

$$x + 3x = 48$$

$$\frac{4x}{4} = \frac{48}{4}$$

$$x = 12$$

Susan is 12

Samantha is 36

Twice a number decreased by 8 is equal to the number increased by 10. Find the number.

Let  $x =$  the number

$$2x - 8 = x + 10$$

$$-x \quad -x$$

$$x - 8 = 10$$

$$\quad +8 \quad +8$$

$$x = 18$$

Patti bought burgers and fries for her children and friends. The burgers cost \$2.05 each and the fries are \$.85 each. She bought a total of 14 items, for a total cost of \$19.10. How many of each did she buy?

Let  $x =$  the number of burgers

$14 - x =$  the number of fries

Note -  $\left(\frac{\text{cost}}{\text{per item}}\right) \times (\# \text{ of items})$  equals the cost for those items

$(2.05)(x)$  equals the cost of the hamburgers

$(.85)(14-x)$  equals the cost of the fries

the total cost was \$19.10 so we can write the equation

$$2.05x + (.85)(14-x) = 19.10$$

$$2.05x + 11.9 - .85x = 19.10$$

$$1.2x + 11.9 = 19.10$$

$$\quad -11.9 \quad -11.9$$

$$\frac{1.2x}{1.2} = \frac{7.2}{1.2}$$

$$x = 6$$

$$14 - x = 14 - 6 = 8$$

6 burgers  
8 fries

6



## Simplify.

$\frac{ -6+4 }{2}$	$\frac{ -6+4 }{2} = \frac{ -2 }{2} = \frac{2}{2} = 1$	- recall the meaning of absolute value
$-5x + x - 6x$	$-5x + x - 6x$ $= -4x - 6x$ $= -10x$	- combining like terms
$(4y-2) - (y+7)$	$5(4y-2) - (y+7)$ $= 20y - 10 - y - 7$ $= 19y - 17$	- distributive property - combining like terms
$\frac{1}{3}(24x-9)$	$\frac{1}{3}(24x-9)$ $= \frac{24}{3}x - \frac{9}{3}$ $= 8x - 3$	- distributive property - reducing fractions
$6a - 3(b-2)$	$6a - 3(b-2)$ $= 6a - 3b + 6$	- distributive property
$(x-7)(2x+3)$	$(x-7)(2x+3) = 2x^2 + 3x - 14x - 21$ $= 2x^2 - 11x - 21$	"foiling"
$(3x-5)^2$	$(3x-5)^2 = (3x-5)(3x-5)$ $= 9x^2 - 15x - 15x + 25$ $= 9x^2 - 30x + 25$	"foiling"
$(5a+6)^2$	$(5a+6)^2$ $= (5a+6)(5a+6)$ $= 25a^2 + 30a + 30a + 36$ $= 25a^2 + 60a + 36$	"foiling"

⑦

Simplify and write answers with positive exponents

$(-2)^2(3)^3$	$(-2)^2(3)^3$ $= 4 \cdot 27$ $= \boxed{108}$	
$(4a^2b)(-3a^5b^3)$	$(4a^2b)(-3a^5b^3)$ $= \boxed{-12a^7b^4}$	Laws of exponents <b>1</b>
$(r^3s^5)^4$	$(r^3s^5)^4$ $= \boxed{r^{12}s^{20}}$	Laws of exponents <b>2 3</b>
$\frac{27x^6y^4}{18x^8y}$	$\frac{27x^6y^4}{18x^8y}$ $= \frac{27}{18} \cdot x^{6-8} \cdot y^{4-1} = \frac{27}{18} x^{-2} y^3 = \boxed{\frac{3y^3}{2x^2}}$	Laws of exponents <b>4 6</b>
$2^{-2} \cdot 4^3$	$2^{-2} \cdot 4^3 = \frac{1}{2^2} \cdot 4^3 = \frac{4^3}{2^2} = \frac{64}{4} = \boxed{16}$	Laws of exponents <b>6</b>
$\frac{y^4 \cdot y^{-2}}{y^{-5}}$	$\frac{y^4 \cdot y^{-2}}{y^{-5}} = \frac{y^2}{y^{-5}} = \frac{y^2 \cdot y^5}{1} = \boxed{y^7}$	Laws of exponents <b>1 7</b>
$\frac{x}{y^2} \div \frac{x}{y^3}$	$\frac{x}{y^2} \div \frac{x}{y^3} = \frac{x}{y^2} \cdot \frac{y^3}{x} = \frac{xy^3}{y^2x} = y^{3-2} = \boxed{y}$	
$\sqrt{64}$	$\sqrt{64} = \boxed{8}$	
$\sqrt{49a^2b^4}$	$\sqrt{49a^2b^4} = \sqrt{49} \cdot \sqrt{a^2} \cdot \sqrt{b^4} = 7abb = \boxed{7ab^2}$	
$\sqrt[3]{27x^{12}}$	$\sqrt[3]{27x^{12}} = \sqrt[3]{27} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3}$ $= 3 \cdot x \cdot x \cdot x \cdot x$ $= \boxed{3x^4}$	

8

# Simplify

$b(-4b^2) + 3b^2(-b)$	$7b(-4b^2) + 3b^2(-b)$ Law 1 $= -28b^3 - 3b^3$ $= \boxed{-31b^3}$
$3x(x+5) - 2(x^2+4x-1)$	$3x(x+5) - 2(x^2+4x-1)$ Law 1 $= 3x^2 + 15x - 2x^2 - 8x + 2$ $= \boxed{x^2 + 7x + 2}$
$2x^4(3x^2 - 2x + 4)$	$2x^4(3x^2 - 2x + 4)$ Law 1 $= \boxed{6x^6 - 4x^5 + 8x^4}$
$\frac{24x^4 - 32x^3 + 16x^2}{8x^2}$	$\frac{24x^4 - 32x^3 + 16x^2}{8x^2} = \frac{24x^4}{8x^2} - \frac{32x^3}{8x^2} + \frac{16x^2}{8x^2}$ Law 2 $= \boxed{3x^2 - 4x + 2}$
$(x^2 - 5x)(2x^3 - 7)$	$(x^2 - 5x)(2x^3 - 7)$ "foiling" $= 2x^5 - 7x^2 - 10x^4 + 35x = \boxed{2x^5 - 10x^4 - 7x^2 + 35x}$
simplify and write answer with positive exponents $\frac{26a^2b^{-5}c^9}{-4a^{-6}bc^9}$	$\frac{26a^2b^{-5}c^9}{-4a^{-6}bc^9} = \frac{26a^{2-(-6)}b^{-5-1}c^{9-9}}{-4}$ Law 4 $= \frac{26a^8b^{-6}c^0}{-4}$ $= \frac{26a^8}{-4b^6}$ Laws 6 8 $= \boxed{-\frac{13a^8}{2b^6}}$ reduce
$(4x^2y)^2(2x^{-3}y)^{-1}$	$(4x^2y)^2(2x^{-3}y)^{-1} = 4^2x^4y^2 \cdot 2^{-1}x^3y^{-1}$ Laws 2 3 $= 4^2x^7y \cdot 2^{-1}$ Law 1 $= \frac{16x^7y}{2}$ Law 6 $= \boxed{8x^7y}$ reduce

# Factor

$3x^2 + 6x$	$3x^2 + 6x$ $= \boxed{3x(x+2)}$	
$x^2 + 6x - 16$	$x^2 + 6x - 16$ $= \boxed{(x+8)(x-2)}$	
$3x^2 + x - 2$	$3x^2 + x - 2$ $= \boxed{(3x-2)(x+1)}$	
$49y^2 + 84y + 36$	$49y^2 + 84y + 36$ $\boxed{(7y+6)(7y+6)} = \boxed{(7y+6)^2}$	
$12x^2 + 12x + 3$	$12x^2 + 12x + 3$ $= 3(4x^2 + 4x + 1)$ $= 3(2x+1)(2x+1) \text{ or } \boxed{3(2x+1)^2}$	
$2x^2 - 5x + 3$	$2x^2 - 5x + 3$ $= \boxed{(2x-3)(x-1)}$	
$6x^2 - x - 15$	$6x^2 - x - 15$ $\boxed{(3x-5)(2x+3)}$	<p>by trial and error we end up with</p> $3x \cdot 2x = 6x^2$ $-5(3) = -15$ $(3x-5)(2x+3)$ $\begin{array}{r} \phantom{3x} \cdot 3x \\ \phantom{3x} \cdot 2x \\ \hline -10x \\ +9x \\ \hline -x \end{array}$ <p><math>-10x + 9x = -x</math> The middle term.</p>

# Solve - Quadratic Equations

$$x^2 - 8x + 7 = 0$$

$$x^2 - 8x + 7 = 0$$

$$(x-7)(x-1) = 0$$

$$\begin{array}{l|l} x-7=0 & x-1=0 \\ +7 & +1 \\ \hline x=7 & x=1 \end{array}$$

$$x^2 + 25 = -10x$$

$$x^2 + 25 = -10x$$

$$+10x \quad +10x$$

$$x^2 + 10x + 25 = 0$$

$$(x+5)(x+5) = 0$$

$$\begin{array}{l|l} x+5=0 & x+5=0 \\ -5 & -5 \\ \hline x=-5 & x=-5 \end{array}$$

get 0 on the right

$$4a^2 + 9a + 2 = 0$$

$$4a^2 + 9a + 2 = 0$$

$$(4a+1)(a+2) = 0$$

$$\begin{array}{l|l} 4a+1=0 & a+2=0 \\ 4a=-1 & a=-2 \\ \hline a=-\frac{1}{4} & a=-2 \end{array}$$

$$9x^2 + 12x + 4 = 16$$

$$9x^2 + 12x + 4 = 16$$

$$-16 \quad -16$$

$$9x^2 + 12x - 12 = 0$$

$$3(3x^2 + 4x - 4) = 0$$

$$3(3x-2)(x+2) = 0$$

$$(3x-2)(x+2) = 0$$

$$\begin{array}{l|l} 3x-2=0 & x+2=0 \\ 3x=2 & x+2=0 \\ \hline x=\frac{2}{3} & x=-2 \end{array}$$

Divide both sides of equation by 3

$$\frac{0}{3} = 0$$

# Simplify Rational Expressions

$\frac{8a^4 - 6a^2 - 2a}{2a^2}$	$\frac{8a^4 - 6a^2 - 2a}{2a^2} = \frac{8a^4}{2a^2} - \frac{6a^2}{2a^2} - \frac{2a}{2a^2} = \boxed{4a^2 - 3 - \frac{1}{a}}$
$\frac{a^2 - 3a - 18}{a^2 - 9}$	$\frac{a^2 - 3a - 18}{a^2 - 9} = \frac{(a+3)(a-6)}{(a+3)(a-3)} = \boxed{\frac{a-6}{a-3}}$
$\frac{x^2 + 7x}{x^2 - 25} \cdot \frac{3x + 15}{x^2}$	$\frac{x^2 + 7x}{x^2 - 25} \cdot \frac{3x + 15}{x^2} = \frac{\cancel{x}(x+7)}{(x-5)(x+5)} \cdot \frac{3(x+5)}{\cancel{x^2}} = \boxed{\frac{3(x+7)}{x(x-5)}}$
$\frac{2}{a-3} \div \frac{5}{2a-6}$	$\frac{2}{a-3} \div \frac{5}{2a-6} = \frac{2}{a-3} \cdot \frac{2a-6}{5} = \frac{2}{\cancel{a-3}} \cdot \frac{2(\cancel{a-3})}{5} = \boxed{\frac{4}{5}}$
$\frac{7}{10y} - \frac{9}{5y^2}$	<div style="display: flex; align-items: center;"> <div style="flex: 1;"> <math display="block">\frac{7}{10y} - \frac{9}{5y^2} = \frac{7y}{10y^2} - \frac{18}{10y^2} = \boxed{\frac{7y-18}{10y^2}}</math> </div> <div style="flex: 1; border-left: 1px solid black; padding-left: 10px;"> <p>LCD = 10y<sup>2</sup></p> <p><math>\frac{7}{10y} \cdot \frac{y}{y} = \frac{7y}{10y^2}</math></p> <p><math>\frac{9}{5y^2} \cdot \frac{2}{2} = \frac{18}{10y^2}</math></p> </div> </div>
$\frac{3}{x} - \frac{5}{x+3}$	<div style="display: flex; align-items: center;"> <div style="flex: 1;"> <math display="block">\frac{3}{x} - \frac{5}{x+3} = \frac{3(x+3)}{x(x+3)} - \frac{5x}{x(x+3)} = \frac{3(x+3) - 5x}{x(x+3)} = \frac{3x+9-5x}{x(x+3)} = \boxed{\frac{9-2x}{x(x+3)}}</math> </div> <div style="flex: 1; border-left: 1px solid black; padding-left: 10px;"> <p>LCD = x(x+3)</p> <p><math>\frac{3}{x} \cdot \frac{(x+3)}{(x+3)} = \frac{3(x+3)}{x(x+3)}</math></p> <p><math>\frac{5}{x+3} \cdot \frac{x}{x} = \frac{5x}{x(x+3)}</math></p> </div> </div>
$\frac{1 + \frac{3}{x}}{x - \frac{9}{x}}$	$\frac{\frac{x}{1} \cdot \left(1 + \frac{3}{x}\right)}{\frac{x}{1} \cdot \left(x - \frac{9}{x}\right)} = \frac{x+3}{x^2-9} = \frac{x+3}{(x+3)(x-3)} = \boxed{\frac{1}{x-3}}$ <p style="text-align: right;">LCD = x</p>

# Lines and Graphs of Lines

Graph and find the slope of this line:

$$3x - 2y = 6$$

$$3x - 2y = 6$$

X	Y
0	-3
2	0

(0, -3) (2, 0)

if  $x=0$   
 $3(0) - 2y = 6$   
 $-2y = 6$   
 $\frac{-2y}{-2} = \frac{6}{-2}$   
 $y = -3$

if  $y=0$   
 $3x - 2(0) = 6$   
 $3x = 6$   
 $\frac{3x}{3} = \frac{6}{3}$   
 $x = 2$

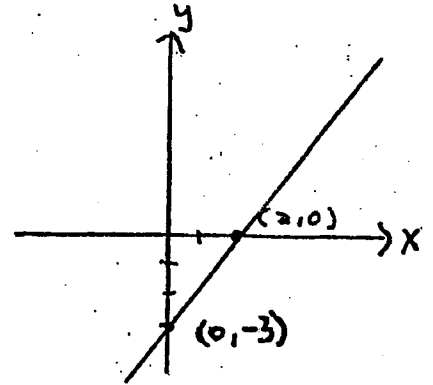
Find slope  
- solve for y

$$3x - 2y = 6$$

$$-3x \quad -3x$$

$$\frac{-2y}{-2} = \frac{-3x + 6}{-2}$$

$$y = \frac{3}{2}x - 3 \quad \text{Slope} = \frac{3}{2}$$



Graph and find the slope of this line:

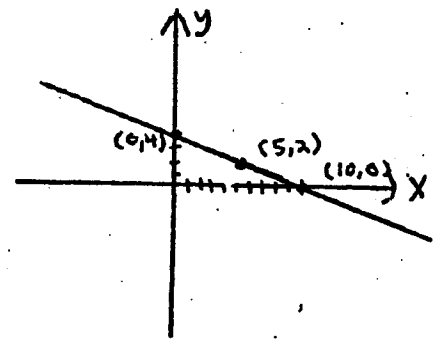
$$y = -\frac{2}{5}x + 4$$

$$y = -\frac{2}{5}x + 4$$

slope =  $-\frac{2}{5}$

y intercept = 4

X	Y
0	4
5	2
10	0



Find the slope of the line that passes thru the points (-2, 7) and (3, -3)

$$(x_1, y_1) (x_2, y_2)$$

$$(-2, 7) (3, -3)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 7}{3 - (-2)} = \frac{-10}{3 + 2} = \frac{-10}{5} = -2$$

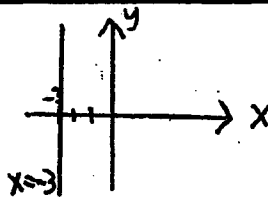
Write an equation of a line whose slope is 2 and whose y intercept is (0, 5)

$$y = mx + b$$

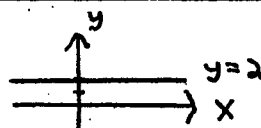
$$y = 2x + 5$$

m = slope  
b = y intercept

Graph  $x = -3$



Graph  $y = 2$



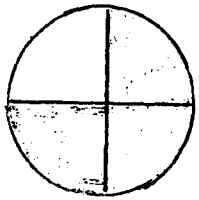
# Systems of Equations; Formulas

<p>Solve the system</p> $2x - 3y = -12$ $x - 2y = -9$	$2x - 3y = -12$ $-2(x - 2y = -9)$ $\frac{2x - 3y = -12}{-2x + 4y = 18}$ $y = 6$ $2x - 3(6) = -12$ $2x - 18 = -12$ $2x = 6$ $x = 3$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">(3, 6)</div>
<p>Solve the system</p> $2x - 3y = -4$ $y = -2x + 4$	$y = -2x + 4$ $2x - 3y = -4$ $+2x \quad +2x$ $-1(2x + y = 4)$ $2x + y = 4$ $\frac{2x - 3y = -4}{-2x - y = -4}$ $-4y = -8$ $y = 2$ $2x - 3(2) = -4$ $2x - 6 = -4$ $2x = 2$ $x = 1$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">(1, 2)</div>
<p>Solve the formula</p> $PV = nRT \text{ for } T$	$\frac{PV}{nR} = \frac{nRT}{nR}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math display="block">\frac{PV}{nR} = T</math> </div> <p>Divide both sides by nR to isolate T</p>
<p>Solve the formula</p> $C = 2\pi r \text{ for } r$	$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math display="block">\frac{C}{2\pi} = r</math> </div> <p>Divide both sides by 2π to isolate r</p>
<p>Solve</p> $y = 3x + 2 \text{ for } x$	$y = 3x + 2$ $-2 \quad -2$ $y - 2 = 3x$ $\frac{y - 2}{3} = \frac{3x}{3}$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <math display="block">\frac{y - 2}{3} = x</math> </div> <p>subtract 2 from both sides to isolate the 3x</p> <p>Divide both sides by 3 to isolate the x</p>



# Fractions

# ARITHMETIC REVIEW



$\frac{3}{4}$  - numerator - how many parts you have (shaded)  
 $\frac{3}{4}$  - denominator - how many parts are in the whole

## Proper Fractions

$$\frac{1}{3}, \frac{5}{6}$$

- the numerator is less than the denominator
- the value of the fraction is less than 1

## Mixed numbers

$$2\frac{1}{5}, 10\frac{3}{4}$$

- consists of a whole number and a proper fraction

## Improper Fractions

$$\frac{16}{7}, \frac{9}{8}, \frac{12}{1}$$

- the numerator is greater than or equal to the denominator.
- the value of the fraction is greater than or equal to 1.

## Changing Mixed Numbers to Improper Fractions

Ex. Change  $2\frac{3}{4}$  to an improper fraction.

$$2 \times 4 = 8$$

$$8 + 3 = 11$$

$$\frac{11}{4}$$

Step 1 multiply the denominator by the whole number.

Step 2 Add the result to the numerator

Step 3 Place the total over the denominator

$$2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4} \text{ answer}$$

## Changing an Improper Fraction to a Mixed Number

Ex. Change  $\frac{13}{5}$  to a mixed number.

$$5 \overline{)13}$$

$$\begin{array}{r} 2 \\ 5 \overline{)13} \\ \underline{-10} \\ 3 \end{array}$$

$$\text{answer } 2\frac{3}{5}$$

Step 1 Divide the numerator by the denominator

Step 2 The whole number part of the answer is 2. The remainder goes over the denominator to make the proper fraction

## Reducing Fractions to Lowest Terms (Simplifying)

Step 1 Find a number that goes evenly into both the numerator and the denominator of the fraction

Step 2 Divide both the numerator and denominator by that number. Repeat until there is no number that can go evenly into both the numerator and the denominator of the fraction.

Ex.  $\frac{6}{8}$  ;  $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$  answer

Ex.  $\frac{24}{36}$  ;  $\frac{24 \div 4}{36 \div 4} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$  answer

Note: If the numerator and the denominator are both even you can start with 2. If not try 3. Using a larger number that goes into both the numerator and the denominator evenly will save steps.

## Multiplying Fractions

Step 1 Change every mixed number to an improper fraction and write every whole number with a denominator of 1.

Step 2 multiply the numerators across, then multiply the denominators across.

Step 3 Reduce the answer. If the answer is an improper fraction you can change it to a mixed number.

Ex.  $\frac{3}{4} \times \frac{5}{6}$   
 $\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} \div 3 = \frac{5}{8}$  answer

Ex.  $2\frac{1}{3} \times 1\frac{3}{5}$   
 $= \frac{7}{3} \times \frac{8}{5} = \frac{56}{15}$  or  $3\frac{11}{15}$  answer

$$\begin{array}{r} 3 \\ 15 \overline{)56} \\ \underline{-45} \\ 11 \end{array}$$

Ex.  $\frac{3}{4} \times 7$   
 $= \frac{3}{4} \times \frac{7}{1} = \frac{21}{4}$  or  $5\frac{1}{4}$  answer

$\frac{5}{8} \times \frac{12}{7}$   
 $= \frac{5}{8} \times \frac{12}{7} = \frac{60}{56} \div 2 = \frac{30}{28} \div 2 = \frac{15}{14}$  or  $1\frac{1}{14}$  answer

## Dividing Fractions

Step 1 Change every mixed number to an improper fraction and write every whole number with a denominator of 1.

Step 2 Change to multiplication and flip the second fraction (the one you are dividing by)

Step 3 multiply the fractions, reduce the answer.

Ex.  $\frac{3}{5} \div \frac{1}{2}$   
 $\frac{3}{5} \times \frac{2}{1} = \frac{6}{5}$  or  $1\frac{1}{5}$  answer

Ex.  $50 \div \frac{1}{4}$   
 $\frac{50}{1} \div \frac{1}{4} = \frac{50}{1} \times \frac{4}{1} = \frac{200}{1} = 200$  answer

Ex.  $2\frac{1}{2} \div 3\frac{1}{8}$   
 $= \frac{5}{2} \div \frac{25}{8} = \frac{5}{2} \times \frac{8}{25} = \frac{40}{50} \div 10 = \frac{4}{5}$  answer

Ex.  $1\frac{1}{5} \div 14$   
 $= \frac{6}{5} \div \frac{14}{1} = \frac{6}{5} \times \frac{1}{14} = \frac{6}{70} \div 2 = \frac{3}{35}$  answer

## Adding and Subtracting Fractions.

You can add and subtract fractions only if they have the same (common) denominators.

A common denominator is a number that can be evenly divided by all the denominators in the problem.

If the denominators are small you can just multiply them together to get a common denominator.

---

Ex.  $\frac{1}{4} + \frac{2}{3}$

Step 1 Find a common denominator

Multiply  $4 \times 3 = 12$ . Use 12 as the common denominator.

$$\begin{array}{r} \frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12} \\ + \frac{2}{3} \cdot \frac{4}{4} = \frac{8}{12} \\ \hline \end{array}$$

Step 2 Rewrite both fractions so that they have the common denominator.

Note: To change the  $\frac{1}{4}$  to  $\frac{3}{12}$  we multiply both the numerator and the denominator of  $\frac{1}{4}$  by 3; since we multiplied  $4 \times 3$  to get the common denominator of 12.

$\frac{11}{12}$  answer

Step 3

add the numerators and write the result over the common denominator.  
Reduce if necessary.

---

Ex.  $\frac{3}{4} - \frac{5}{8}$

Step 1 Find a common denominator

In this example 8 can be divided evenly by 4 so we can use it as the common denominator.

$$\begin{array}{r} \frac{3}{4} \cdot \frac{2}{2} = \frac{6}{8} \\ - \frac{5}{8} \cdot \frac{1}{1} = \frac{5}{8} \\ \hline \end{array}$$

Step 2

Rewrite both fractions so that they have the common denominator.

$\frac{1}{8}$  answer

Step 3

subtract the numerators and write the result over the common denominator.

---

Ex.  $6\frac{1}{2} + 2\frac{4}{5}$

Step 1 Find a common denominator.  $2 \times 5 = 10$

Step 2 Rewrite both fractions so that they have the common denominator.

$$6\frac{1}{2} \cdot \frac{5}{5} = \frac{5}{10}$$

$$+ 2\frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10}$$

Step 3 add the fractions and add the whole numbers

$$8\frac{13}{10} = 8 + 1\frac{3}{10} = 9\frac{3}{10}$$

Step 4 Write the answer as a mixed number. Change  $\frac{13}{10}$  to  $1\frac{3}{10}$  and add it to the 8 to get the final answer.

# Decimals

## Adding and Subtracting Decimals

Step 1 Line up the decimal points

Step 2 Add or subtract

Ex. Add  $12 + 14.03 + 200.4$

$$\begin{array}{r} 12.00 \\ 14.03 \\ + 200.40 \\ \hline 226.43 \end{array}$$

Note  $12 = 12.00$   
- zeros can be added to the right of the decimal point  
- Decimal point in answer is lined up vertically with numbers being added or subtracted

Ex. Subtract  $8.01 - 3.696$

$$\begin{array}{r} 7.910 \\ 8.010 \\ - 3.696 \\ \hline 4.314 \end{array}$$

## Multiplying Decimals

Step 1 multiply the decimals as you would with whole numbers.

Step 2 Add the number of decimal places in each of the numbers being multiplied. Place the decimal point in the answer that total number of spaces from the right.

decimal places - numbers to the right of the decimal point

Ex.  $1.23 \times 0.5$

$$\begin{array}{r} 1.23 \\ \times 0.5 \\ \hline 0.615 \end{array}$$

(2 decimal places)  
(1 decimal place)  
(answer has 3 decimal places)

$\frac{2}{+1}$   
 $\frac{3}{3}$

## Dividing Decimals

### Dividing a decimal by a whole number

Place the decimal point in the answer directly above the decimal point in the problem. Then divide as you would with whole numbers

Ex. Divide  $1.512 \div 72$

$$\begin{array}{r} .021 \\ 72 \overline{) 1.512} \\ \underline{-144} \phantom{0} \\ 72 \\ \underline{-72} \\ 0 \end{array}$$

Note - you must place a zero in your answer to hold the tenths place

### Dividing a decimal by a decimal

Ex. Divide  $0.642 \div 0.03$

$$0.03 \overline{) 0.642}$$

$$\longrightarrow 3 \overline{) 64.2}$$



$$\begin{array}{r} 21.4 \\ 3 \overline{) 64.2} \\ \underline{-6} \phantom{0} \\ 04 \\ \underline{-3} \phantom{0} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

Move the decimal point of the divisor (the number outside the bracket) as far to the right as you can go. Then move the decimal point of the dividend (the number inside the bracket) the same number of places to the right

move the decimal point up  
Then divide as you would with whole numbers

## Percents

Fractions, decimals, and percents are used to describe a part of a whole.

Consider an example drawn from money

- a quarter, 25 out of 100 pennies, 25 cents, .25

<u>Fraction</u>	<u>Decimal</u>	<u>Percent</u>
$\frac{1}{4} = \frac{25}{100}$	0.25	25%

### Changing Decimals to Percents

- move the decimal point 2 places to the right and add the % sign.

Ex.  $0.25 = 25\%$

$0.80 = 80\%$

$0.07 = 7\%$

### Changing Percents to Decimals

- move the decimal point 2 places to the left and drop the % sign.

Ex.  $25\% = 0.25$

$30\% = 0.3$

$42\% = 0.42$

$5\% = 0.05$

$0.2\% = 0.002$

Note in 25% the decimal point is 25% even though it is not written.

### Changing Fractions to Percents

- divide the top number of the fraction by the bottom number (that gives you the equivalent decimal)  
- then move the decimal point 2 places to the right and add a % sign.

ex  $\frac{1}{4}$   $4 \overline{)1.000} \begin{array}{r} .25 \\ -8 \\ \hline 20 \end{array} = .25 = 25\%$

### Changing Percents to Fractions

- Write the percent as a fraction with 100 as the denominator.  
- reduce the fraction.

Ex  $25\% = \frac{25}{100} \div 25 = \frac{1}{4}$

# Solving Percent Problems

Notes- There are key words that will help us solve these problems

The word of means to multiply  $\times$

The word is means it is equal to  $=$

Percents cannot be used in calculations they must be rewritten as decimals.

We will let  $n$  represent the number we are trying to find.

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## Finding The Percent of a number

Ex. What is 25% of 300?

Solution

$$n = 25\% \times 300$$

Comments

- replace what with  $n$   
is with  $=$   
of with  $\times$

$$n = .25 \times 300$$

- convert to decimal  
25%  $\rightarrow$  .25

$$n = 75$$

- multiply

---

## Finding What Percent One Number is of another.

Ex. 32 is what percent of 160?

Solution

$$32 = n \times 160$$

Comments

- replace is with  $=$   
what percent with  $n$   
of with  $\times$

$$\frac{32}{160} = \frac{n \times 160}{160}$$

$$n = \frac{32 \div 8}{160 \div 8} = \frac{4}{20} = \frac{1}{5}$$

- solve equation for  $n$  by dividing by 160

$$n = \frac{1}{5} = 5 \overline{)1.00} = 20\%$$

- Since we are looking for a % we must convert  $\frac{1}{5}$  to a %

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## Finding a Number When a Percent of It is Given.

Ex. 20% of what number is 16?

Solution

$$20\% \times n = 16$$

Comments

- replace of with  $\times$   
what with  $n$   
is with  $=$

$$\frac{.2 \times n}{.2} = \frac{16}{.2}$$

- convert to decimal  
20%  $\rightarrow$  .20 = .2

$$n = \frac{16}{.2} = 80$$

- solve equation for  $n$  by dividing by .2

$$.2 \overline{)16.0} \\ \underline{16} \\ 0$$

# Practice Examples - Fractions

$\frac{1}{3} \div \frac{5}{22}$	$\frac{1}{3} \div \frac{5}{22} = \frac{1}{3} \times \frac{22}{5} = \frac{22}{15}$ or $1\frac{7}{15}$
$1\frac{1}{6} \times 10$	$1\frac{1}{6} \times 10 = \frac{7}{6} \times \frac{10}{1} = \frac{70}{6} \div 2 = \frac{35}{3}$ or $11\frac{2}{3}$
Find $\frac{2}{3}$ of 20.	$\frac{2}{3} \times 20 = \frac{2}{3} \times \frac{20}{1} = \frac{40}{3}$ or $13\frac{1}{3}$ Note: Here <u>the word of</u> means multiply.
) $6\frac{5}{6} + 7\frac{3}{4}$	$6\frac{5}{6} \cdot \frac{2}{2} = \frac{10}{12}$ $+ 7\frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}$ <hr/> $13\frac{19}{12} = 13 + 1\frac{7}{12} = 14\frac{7}{12}$ rewrite $\frac{19}{12}$ as $1\frac{7}{12}$ and add it to 13 to get the mixed number answer
) $7 \div \frac{2}{5}$	$7 \div \frac{2}{5} = \frac{7}{1} \div \frac{2}{5} = \frac{7}{1} \times \frac{5}{2} = \frac{35}{2}$ or $17\frac{1}{2}$
) $11\frac{2}{3} - 2\frac{1}{2}$	$11\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$ $- 2\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$ <hr/> $9\frac{1}{6}$
) A student's tuition was \$3630. A loan was obtained for $\frac{5}{6}$ of the tuition. How much was the loan?	$\frac{5}{6} \times \frac{3630}{1} = \frac{18150}{6} = \$3025$ Note: the word of means multiply.
) A class has 14 men and 16 women. What fraction of the class is men?	$\frac{14 \text{ men} + 16 \text{ women}}{30 \text{ in class}} = \frac{14}{30} = \frac{7}{15}$ 14 - number we are interested in (the men) 30 - number of parts in the whole
) $4\frac{1}{2} \cdot \frac{1}{5}$	$4\frac{1}{2} \cdot \frac{1}{5} = \frac{9}{2} \cdot \frac{1}{5} = \frac{9}{10}$ Note: $\cdot$ means multiply
) $\frac{15}{2} \div 3$	$\frac{15}{2} \div 3 = \frac{15}{2} \div \frac{3}{1} = \frac{15}{2} \times \frac{1}{3} = \frac{15}{6} \div 3 = \frac{5}{2}$ or $2\frac{1}{2}$
) A car traveled 200 mi. on $12\frac{1}{2}$ gal. of gas. How many miles per gallon did it get?	miles per gallon is calculated by dividing. miles $\div$ gallons Divide $200 \div 12\frac{1}{2}$ $= \frac{200}{1} \div \frac{25}{2} = \frac{200}{1} \times \frac{2}{25} = \frac{400}{25} = 16$ miles per gallon

# Practice Examples - Decimals

multiply $0.98 \times 0.7$	$\begin{array}{r} 0.98 \\ \times 0.7 \\ \hline 0.686 \end{array}$ (2 decimal places) (1 decimal place) (answer has 3 decimal places)
Divide $4.6 \div 10$	$\begin{array}{r} 0.46 \\ 10 \overline{) 4.60} \\ \underline{-40} \\ 60 \\ \underline{-60} \\ 0 \end{array}$ or use short cut - dividing by a power of 10 moves the decimal point to the left the same number of places as there are zeros in the power of 10
Divide $1.0698 / 0.001$	$\begin{array}{r} 1069.8 \\ 0.001 \overline{) 1.0698} \\ \underline{-006} \\ 09 \\ \underline{-9} \\ 08 \\ \underline{-08} \\ 00 \end{array}$
Courtney ran the 100-meter relay in 21.4 seconds, and later ran the same distance in 16.7 seconds. By how much did she improve her time?	subtract $21.4$ $- 16.7$ $\hline 4.7 \text{ seconds}$
$124 + 1.3 + 2.07$	$\begin{array}{r} 124.00 \\ 1.30 \\ + 2.07 \\ \hline 127.37 \end{array}$
$31 - 0.9927$	$\begin{array}{r} 31.0000 \\ - 0.9927 \\ \hline 30.0073 \end{array}$
multiply $0.2 \times 0.2$	$\begin{array}{r} 0.2 \\ \times 0.2 \\ \hline 0.04 \end{array}$ 1 decimal place 1 decimal place 2 decimal places
$0.2592 \div 0.8$	$\begin{array}{r} 0.324 \\ 8 \overline{) 2.592} \\ \underline{-24} \\ 19 \\ \underline{-16} \\ 32 \\ \underline{-32} \\ 0 \end{array}$
$2.464 \div 1.12$	$\begin{array}{r} 2.2 \\ 112 \overline{) 2.464} \\ \underline{224} \\ 224 \\ \underline{224} \\ 0 \end{array}$



Practice Problems - Percents - Decimals - Fractions

) 40 is what percent of 800?

$$40 = n \times 800$$

$$\frac{40}{800} = \frac{n \times 800}{800}$$

$$n = \frac{40 \div 10}{800 \div 10} = \frac{4 \div 4}{80 \div 4} = \frac{1}{20} = 20 \overline{) 1.00} = .05 = \boxed{5\%}$$

) Write as a decimal 0.71%

$$\boxed{0.0071}$$

Note - you are given a %  
- move the decimal point 2 places to the left and drop the % sign.

) What is 60.5% of 80?

$$n = 60.5\% \times 80$$

$$n = .605 \times 80$$

change % to decimal

$$\boxed{n = 48.4}$$

) 15 is what percent of 60?

$$15 = n \times 60$$

$$\frac{15}{60} = \frac{n \times 60}{60}$$

$$n = \frac{15 \div 15}{60 \div 15} = \frac{1}{4} = .25 = \boxed{25\%}$$

) A pharmacist has 100ml of a solution of alcohol and water; 5% is alcohol. How many milliliters are alcohol?

Find - What is 5% of 100?

$$n = 5\% \times 100$$

$$n = .05 \times 100$$

$$n = \boxed{5 \text{ ml.}}$$

) Pam earns \$15,000 a year. She gets a 7% raise. What is her new yearly salary?

The raise is 7% of 15,000.

Find - What is 7% of 15000?

$$n = 7\% \times 15000$$

$$n = .07 \times 15000$$

$$n = 1050$$

$$\text{new salary} = 15,000 + 1050 = \boxed{\$16,050}$$

) Write as a decimal  $\frac{7}{25}$

$$25 \overline{) 7.00} \quad \boxed{.28}$$

$$\begin{array}{r} .28 \\ 25 \overline{) 7.00} \\ \underline{50} \phantom{0} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

) A class has 14 men and 6 women. What percent of the class is women?

$$\begin{array}{r} 14 \text{ men} \\ + 6 \text{ women} \\ \hline 20 \text{ total class} \end{array}$$

$$\frac{6}{20} = 20 \overline{) 6.00} = .3 = \boxed{30\% \text{ women}}$$

Practice Problems - percents - Decimals - Fractions

9) 16% of what number is 42?

$$16\% \times n = 42$$

$$.16 \times n = 42$$

$$\frac{.16 \times n = 42}{.16 \quad .16}$$

$$n = 262.5$$

$$\begin{array}{r} 262.5 \\ .16 \overline{)42.000} \\ \underline{-33} \phantom{0} \\ 100 \\ \underline{-96} \phantom{0} \\ 40 \\ \underline{-33} \phantom{0} \\ 70 \\ \underline{-70} \\ 0 \end{array}$$

10) If 12 is 50% of some number, what is 45% of that number?

This is a 2 part problem

First - Find 12 is 50% of what number

$$\frac{12}{.5} = \frac{.5 \times n}{.5}$$

$$n = 24$$

Second - Find what is 45% of 24?

$$n = .45 \times 24$$

$$n = 10.8$$

$$\begin{array}{r} 10.8 \\ .45 \overline{)24} \\ \underline{135} \\ 96 \\ \underline{90} \\ 60 \\ \underline{54} \\ 60 \\ \underline{54} \\ 60 \\ \underline{54} \\ 60 \end{array}$$

11) Write as a fraction. 35%

$$\frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

12) The regular price of a suit is \$102. It is on sale for 35% off. What is the discount?

Find what is 35% of \$102?

$$n = .35 \times 102$$

$$n = \$35.70$$

$$\begin{array}{r} 102 \\ \times .35 \\ \hline 510 \\ 306 \\ \hline 35.70 \end{array}$$

13) In one year the population of Rye Town increased from 900 to 981. What was the percent of the increase?

$\frac{\text{Change in population}}{\text{Original population}} \rightarrow$  changed to a percent

$$\frac{91 \div 9}{900 \div 9} = \frac{9}{100} = .09 = 9\%$$

14) Write 4.15 as a fraction

4.15

↑

5 is in the hundredths place so

$$4.15 = 4 + \frac{15}{100} = 4 \frac{15 \div 5}{100 \div 5} = 4 \frac{3}{20}$$

15) Divide  $6 \overline{)335}$ . Write a mixed number for the answer.

$$\begin{array}{r} 55 \\ 6 \overline{)335} \\ \underline{30} \phantom{0} \\ 35 \\ \underline{30} \\ 5 \end{array}$$

$$55 \frac{5}{6}$$

16) Write 0.125 as a fraction

0.125

↑

5 is in the thousandths place so

$$0.125 = \frac{125 \div 25}{1000 \div 25} = \frac{5 \div 5}{40 \div 5} = \frac{1}{8}$$